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EFFECT OF FGM COATING ON THERMOELASTIC CHARACTERISTICS OF A ROTATING CIRCULAR DISK

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ABSTRACT

This study focuses on the analysis of thermoelastic characteristics of a thin circular disc having a concentric hole and a functionally graded material (FGM) coating at the outer surface. The disc is subjected to a temperature gradient filed and an inertial force due to rotation of the disc. The coating region is assumed to have an exponential variation of all the material properties except the Poisson's ratio which is assumed to be constant throughout the disc. The incompatible eigenstrain developed in the disc as a result of the temperature gradient field and nonuniform coefficient of thermal expansion (CTE) is taken into account. Using the 2-D thermoelastic theories, the problem is reduced to the solution of a second order differential equation which is solved by a finite element model developed based on the variational approach and Ritz method. The finite element model is demonstrated for an Al disc with an Al/Al_2O_3 FGM coating for the analysis of thermoelastic characteristics corresponding to more practical boundary conditions of the disc. The numerical results reveal that the FGM coating thickness is one of the important parameters to be considered in order to design a grinding disc or a cutter with an FGM coating.

Key words: Functionally graded material, Coating, Circular disk, Thermoelasticity, Thermal load, Finite element method.

1. INTRODUCTION

Functionally graded materials (FGMs) are a new generation of engineered materials wherein the microstructural details are spatially varied through nonuniform distribution of the reinforcement phases. The FGM concept was originated in Japan in 1984 during a space plane project, in the form of a proposed thermal barrier material [1].Usually, these materials are made of ceramics and metals to resist severe environmental effects, such as wear, corrosion, and large temperature gradient in one hand, and ensure toughness and thermal conductivity on the other hand. FGMs have been promising candidates for various structural components such as FGM beams [2-4], plates [5-6], and cylinders [7-8] which have been studied under various thermal and mechanical loading conditions. Afsar *et al*. [6] examined the effects of nonhomogeneous parameter on the elastic field in an FGM rectangular plate subjected to a biaxial tensile load. FGM circular cylinders were considered by Obata and Noda [7] and Liew*et al.* [8] to analyze the thermal stresses. FGM circular cylinders were also considered by Afsar and his co-workers [9, 10] for the analysis of brittle fracture characteristics by taking into account the effect of incompatibility of eigenstrain developed in the cylinder due to nonuniform CTE as a result of temperature change.

In the studies mentioned above, attention was focused mainly on the structural performance of FGMs. To investigate the functional performance of these materials, FGMs are widely explored for thermal barrier [11] and tribological [12] coatings. However, the analysis of FGMs as cutting and grinding tools has received only a little attention so far. Cho and Park [13] investigated the thermoelastic characteristics of functionally graded lathe cutting tools composed of Cr–Mo steel and ceramic tip. They demonstrated that an added FGM layer between the steel shank and the ceramic tip relaxes the thermo-mechanical stress concentration. In our previous study [14], we have demonstrated that a circular FGM cutter or grinding disk can be designed with better thermoelastic characteristics if certain parameters, namely, temperature distribution, angular speed, radial thickness, and outer surface temperature, are controlled properly. For the purpose of analyzing the thermoelastic characteristics, a finite element model was developed. However, the FGM disk considered in our previous study [14] had a material gradation throughout the entire radial thickness, i.e., from the inner surface to the outer surface of the disc. Such an FGM disc is, in fact, not feasible due to manufacturing limitations as well as cost. In an attempt to get rid of this problem, an alternate approach is sought to design the disc with a thin FGM coating at the outer surface only instead of an FGM disc having material gradation throughout the entire radial thickness of the disc. To realize this goal, it is an utmost necessity to understand and quantify the thermoelastic characteristics of such a disc. The present study is, therefore, aimed at analyzing the thermoelastic characteristics of a thin circular disc with a thin FGM coating at the outer surface of the disc.

2. MATHEMATICAL MODEL OF THE PROBLEM

The outer surface of a cutter or a grinding disc should be made of such a material that it can withstand high temperature and prevent wear. On the other hand, the inner region of it should be thermally conductive to facilitate cooling and tougher to absorb torsional energy. Using a bimetallic disc can serve the purpose but the unequal coefficient of thermal expansion causes a miss-fit strain resulting in delamination and, consequently, failure of the disc. To get rid of these problems, the disc can be designed with an FGM coating at the outer surface and a homogenous material at the inner portion of the disc as shown in Fig. 1.The disc has a concentric hole of radius *a* with an outer radius *b*. The region (*h*-*a*) consists of homogenous material B only. The coating region *c*= (*b*-*h*) consists of *A/B* FGM where the distribution of materials *A* and *B* varies continuously from *r* = *h* to *b*.

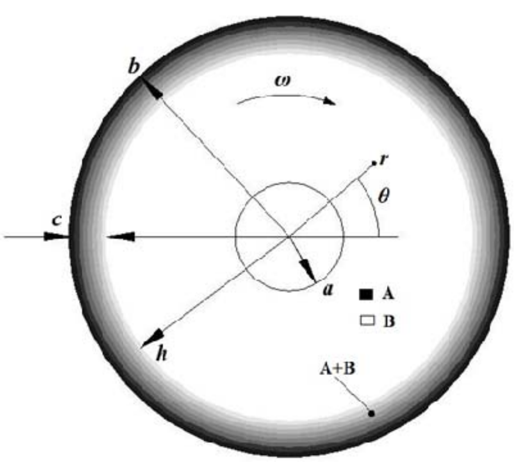

Fig. 1. Schematic diagram of a disc with an FGM coating at the outer surface

The Poisson's ratio is assumed to be constant throughout the entire disc and the Young's modulus (*E*), CTE (*α*), and density (*ρ*) is assumed to vary exponentially as

$$E = E_0 e^{\beta r} \tag{1a}$$

$$\alpha = \alpha_0 e^{\gamma r} \tag{1b}$$

$$\rho = \rho_0 e^{\mu r} \tag{1c}$$

The constants of Eq. (1) are determined as:

$$E_0 = E_B e^{-\beta h} \tag{2a}$$

$$\alpha_0 = \alpha_B e^{-\gamma h} \tag{2b}$$

$$\rho_0 = \rho_B e^{-\mu h} \tag{2c}$$

$$\beta = \frac{1}{b-h} \ln\left(\frac{E_A}{E_B}\right) \tag{3a}$$

$$\gamma = \frac{1}{b-h} \ln\left(\frac{\alpha_A}{\alpha_B}\right) \tag{3b}$$

$$\mu = \frac{1}{b-h} \ln\left(\frac{\rho_A}{\rho_B}\right) \tag{3c}$$

The parameters with subscripts *A* and *B* indicate the properties of the respective constituent materials *A* and *B*. Although Eqs. (1) - (3) express the properties of the FGM coating region ($h \leq r \leq b$), the same expressions can be utilized to determine the properties of the homogeneous region ($a \leq r \leq h$) by setting the values of *β*, *γ*, and *μ* to zero.

3. FORMULATION OF THE PROBLEM

The disc considered in the present study is subjected to a temperature gradient field and an inertia force due to rotation of the disc. The inner surface is assumed to be fixed to a shaft while the outer surface is free from any mechanical loading. Thus, the boundary conditions can be stated as

(i) $r = a$, $u_r = 0$;
(ii) $r = b$, $\sigma_r = 0$;

Due to nonuniform CTE and temperature gradient field, an incompatible eigenstrain [15] is developed in the disc which is given by

$$\varepsilon^* = \propto (r) T(r) \tag{4}$$

Here, *T(r)* is the change in temperature at any point *r* of the disc. By making use of 2-D thermoelastic theories and Eqs. (1) – (4), one readily obtains [14]

$$\frac{d^2F}{dr^2} + \left(\frac{1}{r} - \beta\right)\frac{dF}{dr} + \frac{1}{r}\left(\beta\nu - \frac{1}{r}\right)F = \rho\omega^2 r(\beta r - \mu r - \nu - 3) - E\alpha\left(\gamma T + \frac{dT}{dr}\right) \tag{5}$$

Solution of Eq. (5) gives the value of F which can be used to determine different components of stress, strain, and displacement from the following expressions [14].

$$\sigma_r = \frac{F}{r} \tag{6a}$$

$$\sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2 \tag{6b}$$

$$\varepsilon_r = \frac{1}{E}\left(\frac{F}{r} - \nu\frac{dF}{dr}\right) - \frac{\nu\rho}{E}\omega^2 r^2 + \varepsilon^* \tag{7a}$$

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{dF}{dr} - \frac{\nu F}{r}\right) + \frac{\rho\omega^2 r^2}{E} + \varepsilon^* \tag{7b}$$

$$u_r = \varepsilon_\theta r \tag{8}$$

Since the analytical solution of Eq. (5) is not realistic, a finite element model is developed for the numerical solution of the problem. As all the parameters are function of r only making it a one dimensional problem, the radial domain $\Omega = (a, b)$ of the disc is divided into N number of subdomains $\Omega^e = (r_e, r_{e+1})$, where $e = 1,2,\ldots,N$. Then following the Variational approach and Ritz method Eq. (5)can be reduced to

$$\sum_{j=1}^{2} K_{ij}^e F_j^e = L_j^e \tag{9}$$

where

$$K_{ij}^e = B(\phi_i^e, \phi_j^e)$$

$$L_i^e = l(\phi_i^e)$$

$$F = \sum_{j=1}^{2} F_j^e \phi_j^e$$

$$B(w,F) = \int_{r_e}^{r_{e+1}} \frac{dw}{dr}\frac{dF}{dr}dr - \int_{r_e}^{r_{e+1}} \left(\frac{1}{r} - \beta\right)\frac{dF}{dr}dr - \int_{r_e}^{r_{e+1}} \frac{1}{r}\left(\beta\nu - \frac{1}{r}\right)wF dr$$

$$l(w) = -\int_{r_e}^{r_{e+1}} wf(r)dr + w(r_{e+1})\frac{dF}{dr}(r_{e+1}) - w(r_e)\frac{dF}{dr}(r_e)$$

$$f(r) = \rho\omega^2 r(\beta r - \mu r - \nu - 3) - E\alpha\left(\gamma T + \frac{dT}{dr}\right)$$

Equation (9) is a system of algebraic equation, which can be used to form a global system of algebraic equation by satisfying the continuity condition $F_2^e = F_1^{e+1}$. Once values of F_j^e is known at the global node points, the components of stress, strain, and displacement are obtained as:

$$\sigma_\mathrm{r} = \frac{1}{\mathrm{r}}\sum_{j=1}^{2} F_j^e \phi_j^e \tag{10a}$$

$$\sigma_\theta = \sum_{j=1}^{2} F_j^e \frac{d\phi_j^e}{dr} + \rho\omega^2 r^2 \tag{10b}$$

$$\varepsilon_r = \frac{1}{E}\sum_{j=1}^{2}\left[\frac{F_j^e\phi_j^e}{r} - \nu F_j^e\frac{d\phi_j^e}{dr}\right] - \frac{\nu\rho\omega^2 r^2}{E} + \varepsilon^* \tag{11a}$$

$$\varepsilon_\theta = \frac{1}{E}\sum_{j=1}^{2}\left[F_j^e\frac{d\phi_j^e}{dr} - \frac{\nu}{r}F_j^e\phi_j^e\right] + \frac{\nu\rho\omega^2 r^2}{E} + \varepsilon^* \tag{11b}$$

$$u_r = \frac{r}{E}\sum_{j=1}^{2}\left[F_j^e\frac{d\phi_j^e}{dr} - \frac{\nu}{r}F_j^e\phi_j^e\right] + \frac{\nu\rho\omega^2 r^3}{E} + \varepsilon^* r \tag{12}$$

4. NUMERICAL RESULTS AND DISCUSSION

The finite element model developed in the present study is applied to an Al disc with an Al/Al_2O_3 FGM coating at the outer surface. The materials *A* and *B* mentioned earlier correspond to Al_2O_3 and Al, respectively. The required properties are presented in Table 1.

From the test of convergence, it is found that the element size of 1 mm ensures the convergence of the results. Therefore, all the numerical results present in this section correspond to element size of 1 mm. To observe the effect of temperature distribution profiles on stress, strain, and displacement components, five different temperature profiles as a function of

Table 1. Mechanical and thermal properties of Al and Al_2O_3

Materials	Properties		
	Young's modulus (Mpa)	Coefficient of thermal Expansion(^0C)	Density (gm/cm^3)
Al	**71**	**23.1×10^{-6}**	**2.70**
Al_2O_3	380	8.0×10^{-6}	0.96

normalized radial distance are considered as shown in Fig. 2. For uniform temperature profile, we see that the radial stress is compressive almost over the entire region of the disc except some inner portion of it. Also, for linear and parabolic2 distribution profiles of temperature, some outer region including the FGM coating experiences the compressive radial stress. For other profiles of temperature distribution, radial stress is positive over the entire region. For all profiles of temperature distribution, radial stress is zero at the outer surface which satisfies the boundary condition of the problem.

Circumferential stress as shown in Fig. 4 has different characteristics than the radial one. For all profiles of temperature distribution, stress is compressive at some part of the disc and tensile at other portion of it. For the uniform profile of temperature distribution, stress is negative whose value remains almost constant over the inner portion of the disc except the small outer region. Here, the stress changes from negative to positive value and continues to increase until the outer surface. For other temperature profiles except the exponential one, stress was positive in about 50% of the inner portion of the disc and followed the trend of uniform one. On the other hand, the exponential temperature distribution yields stress whose magnitude is the minimum at almost entire region of the disc. However from the beginning of the coating, the stress changes to compressive one whose value continues to increase until the outer surface of the disc.

The effect of temperature distribution on the displacement component is depicted in Fig. 5. As seen, the displacement is zero at the inner surface satisfying the boundary condition and gradually increases to the maximum value at the outer surface for all the temperature profiles. However, the gradient of the displacement curves is smaller at the coating region. The variation of displacement is the minimum for exponential temperature profile.

To see the effect of angular speed on thermoelastic characteristics of the disc, four different speeds are considered and the results are presented in Figs. 6 to 8. As the inertial force is proportional to the square of the angular speed, and in general, the stress, strain, and displacement are proportional to the force, the magnitude of stress and displacement increases with the increase of the speed as seen from the figures.

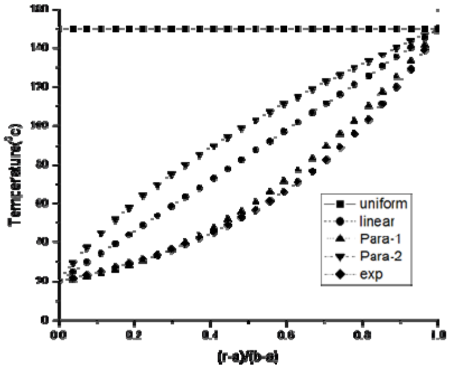

Fig. 2. Prescribed distribution of temperature profiles

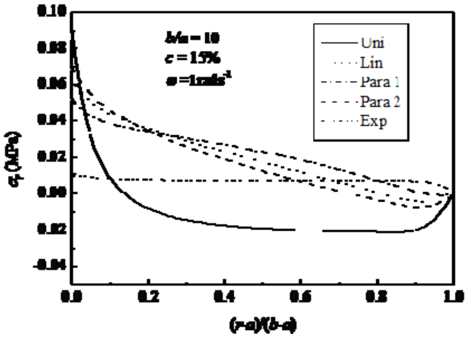

Fig 3. Effect of temperature distribution on radial stress

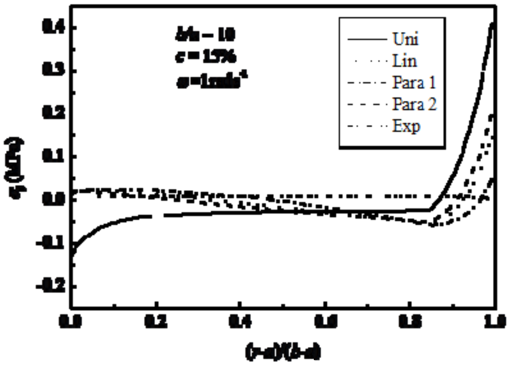

Fig. 4. Effect of temperature distribution on circumferential stress

The radial stress exhibits a slightly different characteristic in the coating region. Here, this stress component does not vary so much with the speed as well as with the radial distance.

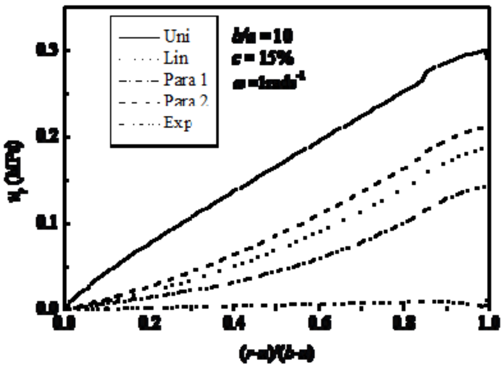

Fig. 5. Effect of temperature distribution on displacement

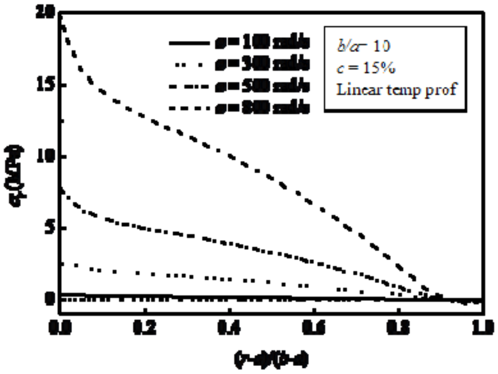

Fig. 6. Effect of angular speed on radial stress

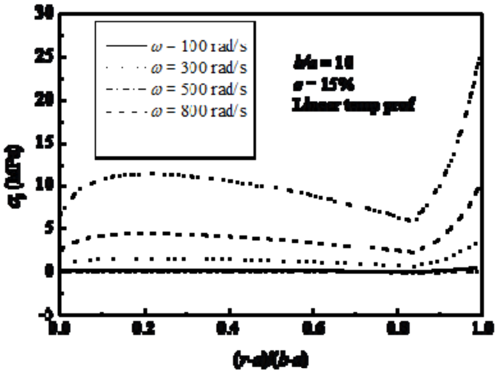

Fig.7. Effect of angular speed on circumferential stress

The effect of FGM coating thickness is illustrated in Figs. 9 to 11. Fig.9 shows that over the major portion of the disc, the radial stress is tensile and its magnitude increases with the decrease of coating thickness. But the coating region experiences compressive stress whose magnitude and the span of the region of compressive stress increase with the increase of the thickness. Circumferential stress as shown in Fig. 10 has the similar trend as that of the radial one but the feature is different at the interface

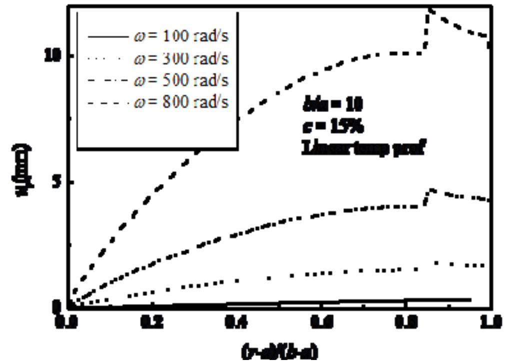

Fig. 8. Effect of angular speed on displacement

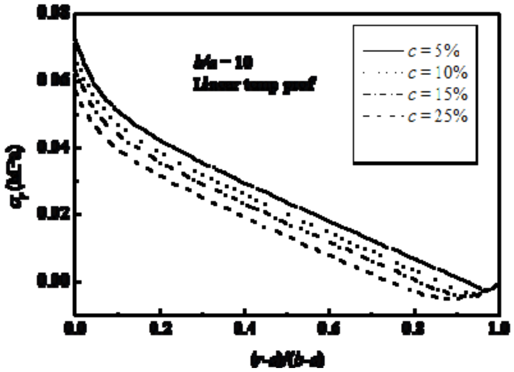

Fig. 9. Effect of coating thickness on radial stress

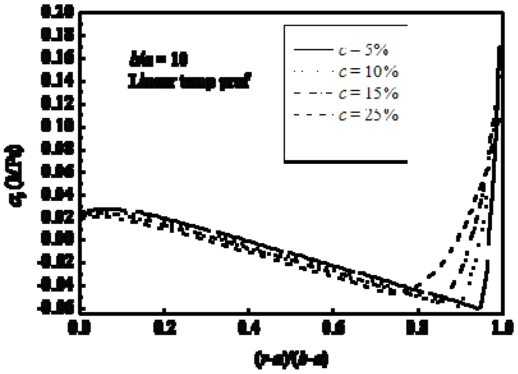

Fig. 10. Effect of coating thickness on circumferential stress

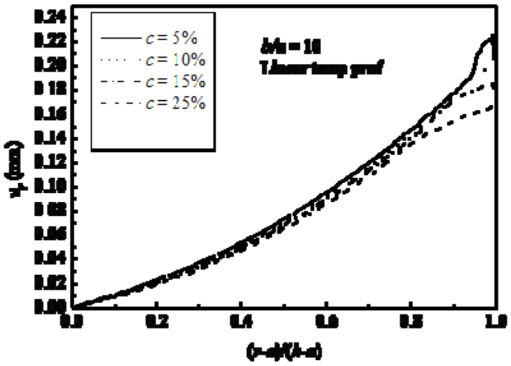

Fig. 11. Effect of coating thickness on displacement

of the coating and homogeneous part of the disc. From Fig. 11, we see that the magnitude of the displacement component decreases with the increase of the coating thickness.

5. CONCLUSION

In the present study, a finite element model has been developed for a rotating circular disc with an FGM coating at the outer surface for the analysis of thermoelastic characteristics of the disc. The Finite element model developed in the study is applicable to an FGM disc, disc with an FGM coating at the outer surface, and also for a homogeneous disc by setting the parameters $\beta = \gamma = \mu = 0$. From the study, it is established that temperature distribution profile, angular speed, and FGM coating thickness are the key parameters to characterize the thermoelastic behavior of the disc. The numerical results presented in the study are helpful to design an actual cutter or grinding disc with an FGM coating.

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