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EFFECTS OF TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY ON MHD FREE CONVECTION FLOW ALONG A VERTICAL FLAT PLATE WITH HEAT GENERATION AND JOULE HEATING

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ABSTRACT

Magnetohydrodynamic (MHD) free convection flow of an electrically conducting fluid along a vertical flat plate with temperature dependent thermal conductivity, heat generation and joule heating effects is analyzed here .The governing equations with associated boundary conditions for this phenomenon are converted to dimensionless forms using a suitable transformation. The transformed non-linear equations are then solved using the implicit finite difference method with Keller-box scheme. Numerical results of the velocity and temperature profiles, skin friction coefficient and surface temperature for different values of the magnetic parameter, thermal conductivity variation parameter, Prandtl number, heat generation and joule heating parameters are presented graphically. Detailed discussion is given for the effects of the aforementioned parameters.

Key words: magnetohydrodynamics, free convection, thermal conductivity variation, heat generation, joule heating, finite difference method.

1. INTRODUCTION

Electrically conducting fluid flow in presence of magnetic field and the effect of temperature dependent conductivity on MHD flow and heat conduction problems are important from the technical point of view and such types of problems have received much attention by many researchers.

Magnetohydrodynamics is that branch of science,

which deals with the motion of highly conducting ionized (electric conductor) fluid in presence of magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field and the action of the magnetic field on these currents give rise to mechanical forces, which modify the fluid. It is possible to attain equilibrium in a conducting fluid if the current is parallel to the magnetic field. Then the magnetic forces vanish and the equilibrium of the gas is the same as in the absence of magnetic fields. In the case when the conductor is either a liquid or a gas, electromagnetic forces will be generated which may be of the same order of magnitude as the hydrodynamical and inertial forces. Thus the equation of motion as well as the other forces will have to take these electromagnetic forces into account.

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfer.

Convection is the transfer of heat by the actual movement of the warmed matter. It is also the transfer of heat energy in a gas or liquid by movement of currents. Considerable convection is responsible for making macaroni rise and fall in a pot of heated water. The warmer portions of the water are less dense and therefore, they rise. Mean while, the cooler portions of the waterfall because they are denser. Conduction is most effective in solids but it can happen in fluids also.

In electronics in particular and in physics broadly, Joule heating is the heating effect of conductors carrying currents. It refers to the increase in temperature of a conductor as a result of resistance to an electrical current flowing through it. At an atomic level, Joule heating is the result of moving electrons colliding with atoms in a conductor, whereupon momentum is transferred to the atom, increasing its kinetic or vibrational energy. Joule heating is caused by interactions between the moving particals that form current and the atomic ions that make up the body of the conductor. It is the process by which the passage of an electric current through a conductor releases heat. Joule's first law is also known as Joule effect. It states that heat generation by a constant current through a resistive conductor for a time whose unit is joule. Heat generation is the ability to emit greater-than-normal heat from the body. Joule heating is the predominant heat mechanism for heat generation in integrated circuits and is an undesired effect.

Model studies of the free and mixed convection flows have earned reputations because of their applications in geophysical, geothermal and nuclear engineering problems. Hossain [1] analyzed the viscous and Joule heating effects on MHD free convection flow with variable plate temperature. Rahman et al. [2] investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction.Rahman and Alim [3] analyzed numerical study of MHD free convective heat transfer flow along a vertical flat plate with temperature dependent thermal conductivity. Nasrin and Alim [4] studied the combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with conduction and joule heating along a vertical flat plate. Alim et.al. [5] analyzed the combined effect of viscous dissipation & joule heating on the coupling of conduction & free convection along a vertical flat plate. Alim et al. [6] investigated Joule heating effect on the coupling of conduction with MHD free convection flow from a vertical flat plate.

The present study is to incorporate the idea of the effects of temperature dependent thermal conductivity on MHD free convection boundary layer flow along a vertical flat plat with heat generation and joule heating. The governing boundary layer equations are transformed into a non-dimentional form and the resulting non-linear system of partial differential equations is reduced to local non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are then solved numerically. Numerical results of the velocity, temperature, local skin friction coefficient and surface temperature distribution for the magnetic parameter, thermal conductivity variation parameter, Prandtl number, heat generation and joule heating parameters are presented graphically.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

At first we consider a steady two-dimensional laminar natural convection flow of an electrically conducting, viscous and incompressible fluid along a vertical flat plate of length l and thickness b (Figure-1). It is assumed that the temperature at the outer surface of the plate is maintained at a constant temperature T_b, where $T_b > T_\infty$, the ambient temperature of the fluid. A uniform magnetic field of strength H_0 is imposed along the $\bar{y}$ -axis i.e.normal direction to the surface and $\bar{x}$ -axis is taken along the flat plate. The coordinate system and the configaration are shown in Figure-1.

The governing equations of such laminer flow with heat generation joule heating and also thermal conductivity variation along a vertical flat plate under the Boussinesq approximations for the present problem for continuity, momentum and energy take the following form

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T_f - T_\infty) - \frac{\sigma H_0^2 \bar{u}}{\rho} \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho c_p}\frac{\partial}{\partial \bar{y}}\left(\kappa_f \frac{\partial T_f}{\partial \bar{y}}\right) + \frac{Q_0}{\rho C_p}(T_f - T_\infty) + \sigma\frac{H_0^2 \bar{u}^2}{\rho C_p} \tag{3}$$

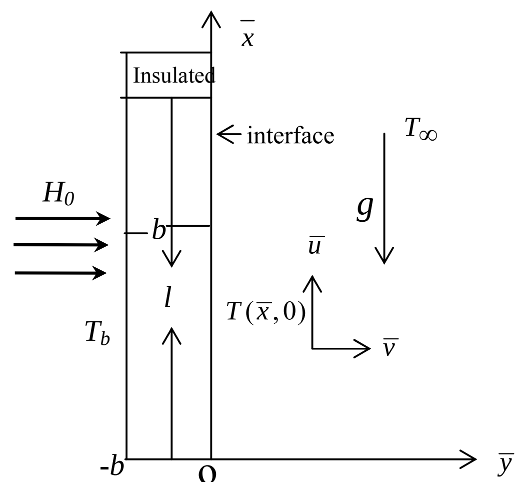

Figure 1: Physical model and coordinate system

Here β is coefficient of volume expansion. The temperature dependent thermal conductivity, which is used by Rahman [2] as follows

$$\kappa_f = \kappa_\infty[1 + \delta(T_f - T_\infty)] \tag{4}$$

where κ_∞ is the thermal conductivity of the ambient fluid and δ is a constant, defined as $\delta = \frac{1}{\kappa_f}\left(\frac{\partial \kappa}{\partial T}\right)_f$. The appropriate boundary condition to be satisfied by the above equations are

$$\left.\begin{array}{l}\bar{u}=0,\ \bar{v}=0\\ T_f=T(\bar{x},0),\ \dfrac{\partial T_f}{\partial \bar{y}}=\dfrac{\kappa_s}{b\kappa_f}(T_f-T_b)\end{array}\right\} on\ \bar{y}=0,\ \bar{x}>0 \qquad (5)$$

$$\bar{u}\to 0,\ T_f\to T_\infty\ \ as\ \bar{y}\to\infty,\ \bar{x}>0$$

The non-dimensional governing equations and boundary conditions can be obtained from equations (1)- (3) using the following dimensionless quantities

$$x=\frac{\bar{x}}{l},\ \ y=\frac{\bar{y}}{l}Gr^{\frac{1}{4}},\ u=\frac{\bar{u}l}{\upsilon}Gr^{-\frac{1}{2}},\ v=\frac{\bar{v}l}{\upsilon}Gr^{-\frac{1}{4}}, \qquad (6)$$

$$\theta=\frac{T_f-T_\infty}{T_b-T_\infty},\ Gr=\frac{g\beta l^3(T_b-T_\infty)}{\upsilon^2}$$

where l is the length of the plate, Gr is the Grashof number, θ is the dimensionless temperature. Now from equations (1)-(3), we get using the following dimensionless equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (7)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \qquad (8)$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2}+\frac{\gamma}{\Pr}\left(\frac{\partial\theta}{\partial y}\right)^2+Ju^2+Q\theta \quad (9)$$

where $\Pr=\dfrac{\mu\ c_p}{\kappa_\infty}$ is the Prandtl number, $M=\dfrac{\sigma\ H_0{}^2l^2}{\mu\ Gr^{1/2}}$ is the dimensionless magnetic parameter, $\gamma=\delta(T_b-T_\infty)$ is the non-dimensional thermal conductivity variation parameter, $J=\dfrac{\sigma H_0^2\nu Gr^{1/2}}{\rho C_p(T_b-T_\infty)}$ is the dimensionless joule heating parameter and $Q=\dfrac{Q_\circ l^2}{\mu C_p Gr^{1/2}}$ is the non-dimensional heat generation parameter.The corresponding boundary conditions (5) then take the following form

$$u=0, v=0,\ \theta-1=(1+\gamma\theta)\ p\frac{\partial\theta}{\partial y}\quad on\ y=0, x>0 \qquad (10)$$

$$u\to 0,\theta\to 0\quad as\ y\to\infty, x>0$$

here $p=\left(\dfrac{\kappa_\infty\ \ b}{\kappa_s l}\right)Gr^{\frac{1}{4}}$ is the conjugate conduction parameter. In the present work we considered p = 1.

To solve the equations (8) and (9) subject to the boundary conditions (10) the following transformations are proposed by Merkin & Pop [7]

$$\psi=x^{4/5}(1+x)^{-1/20}f(x,\eta)$$

$$\eta=yx^{-1/5}(1+x)^{-1/20} \qquad (11)$$

$$\theta=x^{1/5}(1+x)^{-1/5}h(x,\eta)$$

Here η is the similarity variable and ψ is the non-dimensional stream function which satisfies the continuity equation and is related to the velocity components in the usual way as

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x}.$$

Moreover, h (x,η) represents the dimensionless temperature. The momentum and energy equations are transformed for the new coordinate system as

$$f'''+\frac{16+15x}{20(1+x)}ff''-\frac{6+5x}{10(1+x)}f'^2$$
$$-Mx^{\frac{2}{5}}(1+x)^{\frac{1}{10}}f'+h=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \qquad (12)$$

$$\frac{1}{\Pr}h''+\frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}hh''+\frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}h'^2$$
$$+\frac{16+15x}{20(1+x)}fh'-\frac{1}{5(1+x)}f'h=\left(f'\frac{\partial h}{\partial x}-h'\frac{\partial f}{\partial x}\right) \qquad (13)$$

where prime denotes partial differentiation with respect to η. The boundary conditions in equation (10) then take the form

$$f(x,0)=f'(x,0)=0$$
$$h'(x,0)=\frac{x^{1/5}(1+x)^{-1/5}h(x,0)-1}{(1+x)^{-1/4}+\gamma x^{1/5}(1+x)^{-9/20}h(x,0)} \qquad (14)$$
$$f'(x,\infty)\to 0, h(x,\infty)\to 0$$

For the numerical computation, it is important to calculate the values of the surface shear stress in terms of the local skin frication coefficient C_{fx}. This can be written in the dimensionless form as Molla et al. [8]

$$C_f=\frac{Gr^{-\frac{3}{4}}l^2}{\mu\upsilon}\tau_w \qquad (15)$$

where $\tau_w[=\mu(\partial\bar{u}/\partial\bar{y})_{\bar{y}=0}]$ is the shearing stress. Using the new variables described in (6), the local skin friction co-efficient can be written as

$$C_{fx}=x^{\frac{2}{5}}(1+x)^{-\frac{3}{20}}f''(x,0) \qquad (16)$$

The numerical values of the surface temperature distribution θ(x, 0) are obtained form the relation

$$\theta(x,0)=x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h(x,0) \qquad (17)$$

3. METHOD OF SOLUTION

This paper investigates the effect of the temperature dependent thermal conductivity on electrically conducting fluid in free convection flow along a vertical flat plate with heat generation and joule heating for strong magnetic field. Along with the boundary conditions (14), the numerical solutions of the parabolic non-linear partial differential equations (12) and (13) will be found by using very efficient implicit finite difference method together with Keller-box elimination technique [9] which is well documented by Cebeci and Bradshaw [10].

4. RESULT AND DISCUTION

The main objective of the present work is to analyze the effect of thermal conductivity variation due to temperature on MHD free convection flow along a vertical flat plate in presence of heat generation and joule heating. In the simulation the values of the Prandtl number Pr are considered to be 0.733, 1.099, 1.63 and 2.18 that correspond to hydrogen, water, glycerin and sulfur dioxide respectively.

The velocity, the temperature, the local skin friction cofficient and the surface temperature distribution profiles obtained from the solutions of equations (12) and (13) are depicted in Figures 2 to 11.Numerical computations are carried out for a range of magnetic parameter M = 0.01, 0.15, 0.30, 0.45, thermal conductivity variation parameter γ = 0.01, 0.25, 0.50, 0.75, heat generation parameter Q = 0.01, 0.05, 0.10, 0.15 and joule heating parameter J = 0.01, 0.20, 0.40, 0.60.

The interaction of the magnetic field and moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion.

In Fig. 2(a), it is shown that the magnetic field action along the horizontal direction retards the fluid velocity with γ = 0.01, Q = 0.01, J = 0.01, and Pr = 0.733. Here position of peak velocity moves toward the interface with the increasing M. From fig. 2(b), it can be observed that the temperature within the boundary layer increases for increasing values of M.

The effect of thermal conductivity variation parameter γ on the velocity and temperature profiles against η within the boundary layer with M = 0.01, Q = 0.01, J = 0.01 and Pr = 0.733 are shown in fig. 3(a) and 3(b), respectively. It is seen from fig. 3(a) and 3(b) that the velocity and temperature increase within the boundary layer with the increasing value of γ. It means that both the velocity boundary layer and the thermal boundary layer thickness increase for large values of γ.

Figure 4(a) and 4(b) illustrate the velocity and temperature profiles against η for different values of Prandtl number Pr with M = 0.01, Q = 0.01, J = 0.01 and γ = 0.01. From fig. 4(a), it can be observed that the velocity decreases as well as its position moves toward the interface with the increasing Pr. From fig. 4(b), it is seen that the temperature profiles shift downword with the increasing Pr.

Figure 5(a) and 5(b) describe the velocity and temperature profiles against η for different values of heat generation parameter Q with M = 0.01, Pr = 0.733, J = 0.01 and γ = 0.01. From Figure 5(a), it can be observed that the velocity increases as well as its position moves toward the interface with the increasing Q. From Figure 5(b), it is seen that the temperature profiles behave the same as increasing within the boundary layer. It means that the velocity boundary layer and the thermal boundary layer thickness increase for large values of Q.

The effect of joule heating parameter J on the velocity and temperature profiles against η within the boundary layer with M = 0.01, Pr = 0.733, Q = 0.01 and γ = 0.01 are shown in Figure 6(a) and 6(b) respectively. The velocity and temperature increase within the boundary layer with the increasing values of J.

The variation of the local skin friction coefficient and surface temperature distribution against x for different values of M with Pr = 0.733, Q = 0.01, J = 0.01 and γ = 0.01 at different positions are illustrated in Figure 7(a) and 7(b), respectively. It is observed from Figure. 7(a) that the increased value of the Magnetic parameter M leads to a decrease the skin friction factor. Again Figure 7(b) shows that the surface temperature increases due to the increased value of the magnetic parameter M. It can also be seen that the surface temperature increases along the upward direction of the plate for a particular M. The magnetic field acts against the flow and reduces the skin friction and produces the temperature at the interface.

Figure 8(a) and 8(b) illustrate the effect of the thermal conductivity variation parameter γ on the skin friction and surface temperature against x with M = 0.01, Q = 0.01, J = 0.01 and Pr = 0.733. It is seen that the skin friction increases monotonically along the upward direction of the plate for a particular values of γ. It is also seen that that the skin friction increases for the increasing γ. The same result is observed for the surface temperature from Figure 8(b). This is to be expected because the higher value for the thermal conductivity variation parameter accelerates the fluid flow and increases the temperature as mentioned in Figure 3(a) and 3(b), respectively.

Figure 9(a) and 9(b) deal with the effect of Prandtl number Pr on the skin friction and surface temperature against x with M = 0.01, J = 0.01, γ = 0.01 and Q = 0.01.It can be observed from fig. 9(a) that the skin frication increases monotonically for a particular value of Pr. It can also be noted that the skin friction coefficient decreases for the increasing values of Pr. From Figure 9(b), it can be seen that the surface temperature decreases due to the Pr increases along the positive x direction for a particular Pr.

Figure 10(a) and 10(b) deal with the effect of Q on the skin friction and surface temperature against x with M = 0.01, J = 0.01 and γ = 0.01 and Pr = 0.733. It can be noted that the skin friction and surface temperature profile increase for the increasing values of Q.

Figure 11(a) and 11(b) deal with the effect of J on the skin friction and surface temperature against x with M = 0.01, Q = 0.01,and γ = 0.01 and Pr = 0.733. It can be noted that the skin friction and surface temperature profiles increase for the increasing values of J.

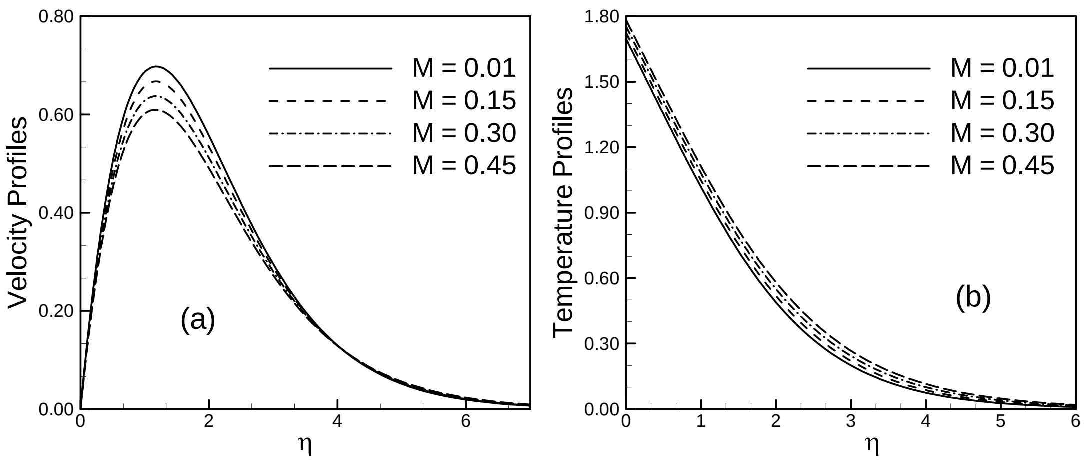

Figure 2(a) Velocity and (b) Temperature profiles against η for varying of M with Pr = 0.733, Q = 0.01, γ = 0.01 and J = 0.01.

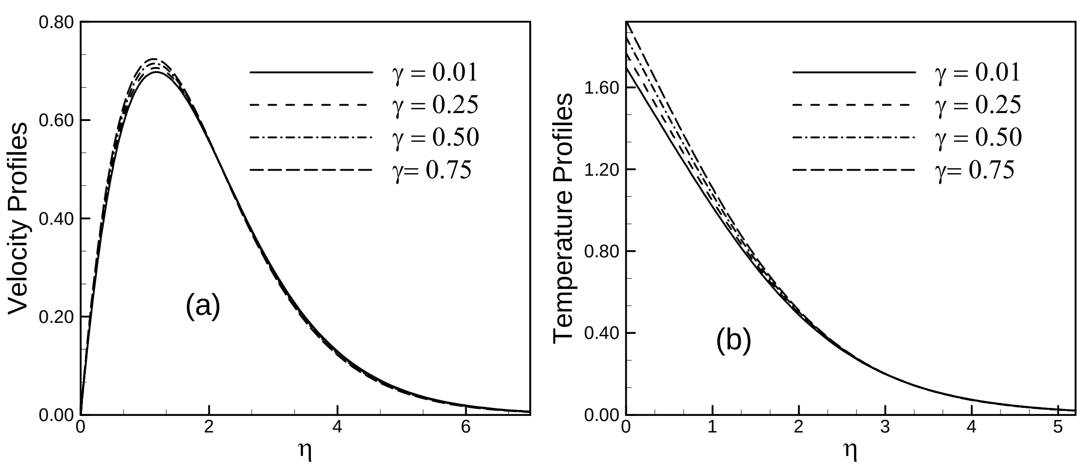

Figure 3(a) Velocity and (b) Temperature profiles against η for varying of γ with Pr = 0.733, Q = 0.01, M = 0.01 and J = 0.01.

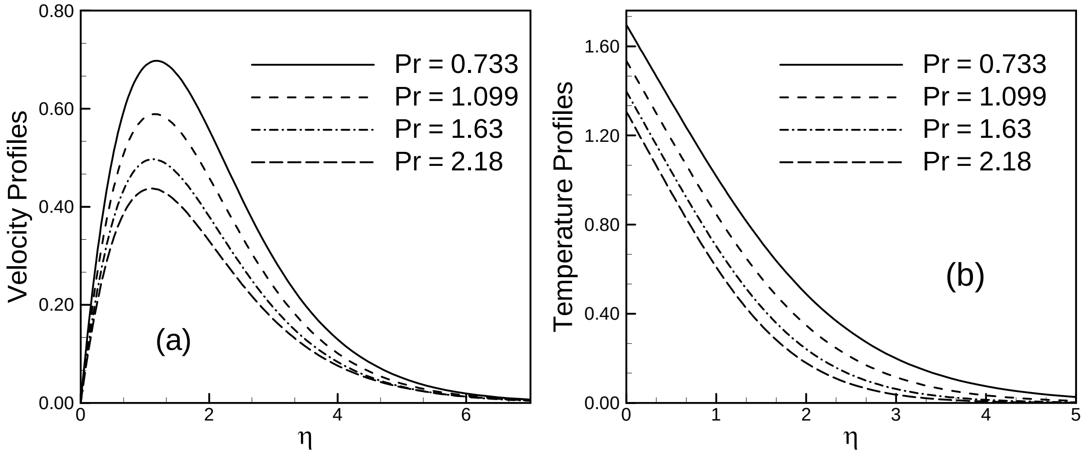

Figure 4(a) Velocity and (b) Temperature profiles against η for varying of Pr with M = 0.01, Q = 0.01, γ = 0.01 and J = 0.01.

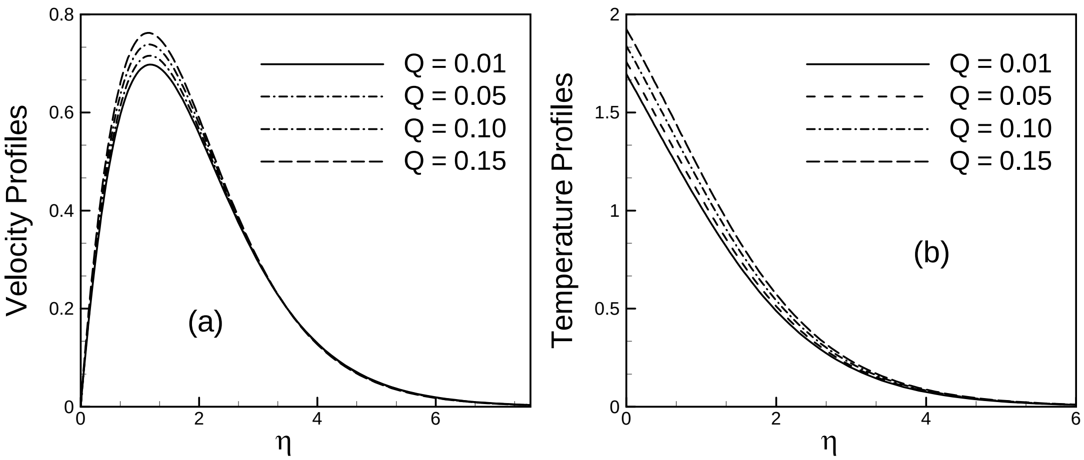

Figure 5(a) Velocity and (b) Temperature profiles against η for varying of Q with M = 0.01, Pr = 0.733, γ = 0.01 and J = 0.01.

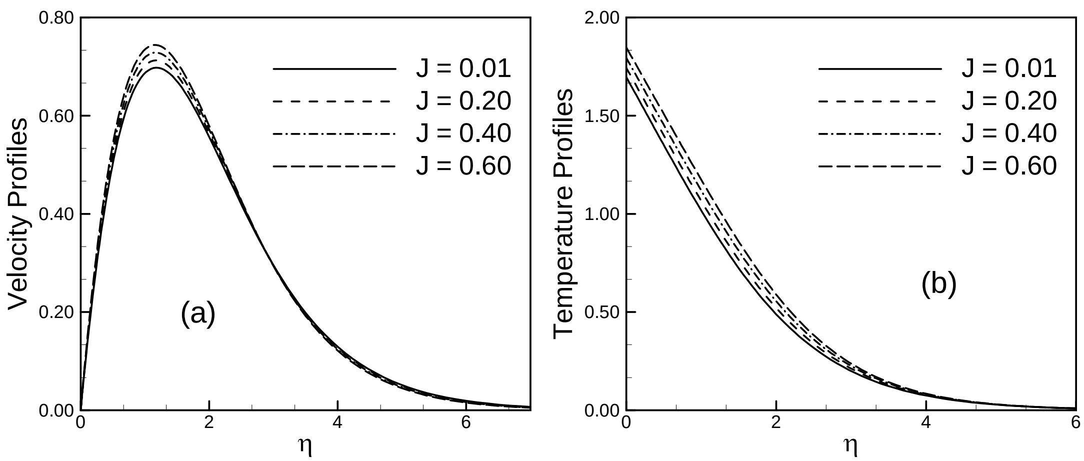

Figure 6(a) Velocity and (b) Temperature profiles against η for varying of J with M = 0.01, Q = 0.01, γ = 0.01 and Pr = 0.733.

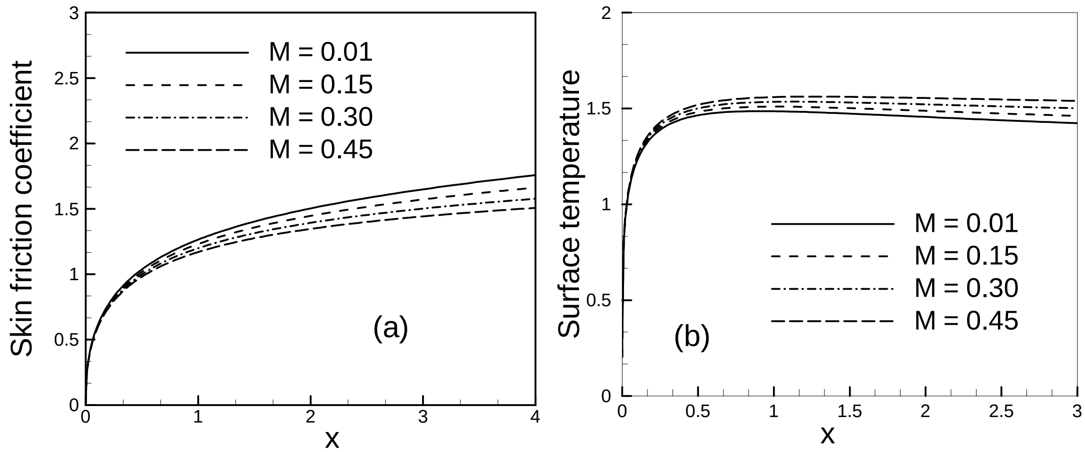

Figure 7(a) Local skin friction coefficient C_{fx} and (b) Surface temperature distribution θ(x, 0) against x for varying of M with Pr = 0.733, Q = 0.01, J = 0.01 and γ = 0.01.

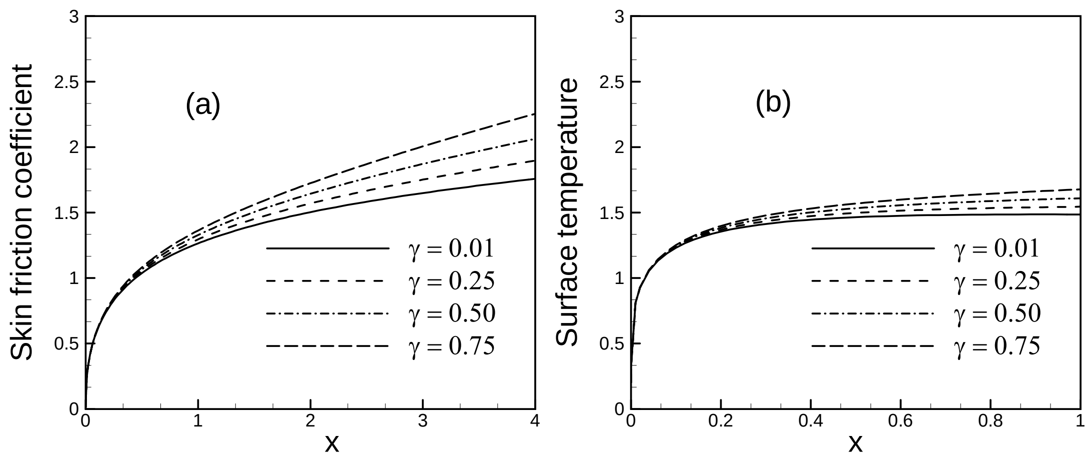

Figure 8(a) Local skin friction coefficient C_{fx} and (b) Surface temperature distribution θ(x, 0) against x for varying of γ with Pr = 0.733, Q = 0.01, J = 0.01 and M = 0.01.

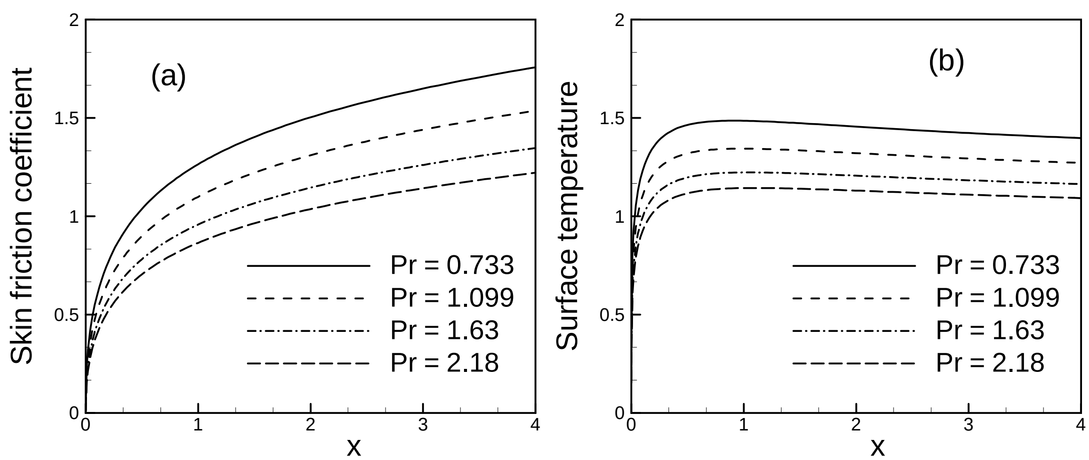

Figure 9(a) Local skin friction coefficient C_{fx} and (b) Surface temperature distribution θ(x, 0) against x for varying of Pr with M = 0.01, Q = 0.01, J = 0.01 and γ = 0.01.

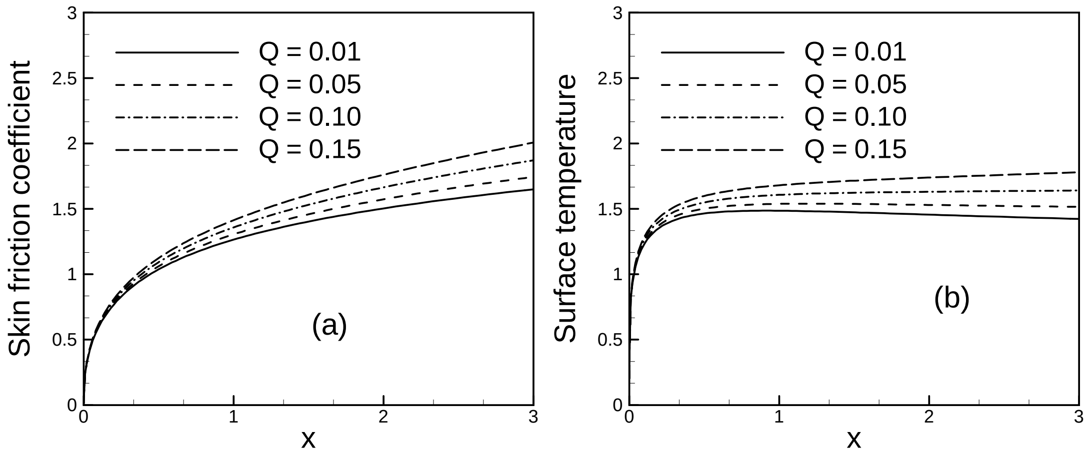

Figure 10(a) Local skin friction coefficient C_{fx} and (b) Surface temperature distribution θ(x, 0) against x for varying of Q with Pr = 0.733, J = 0.01, M = 0.01 and γ = 0.01.

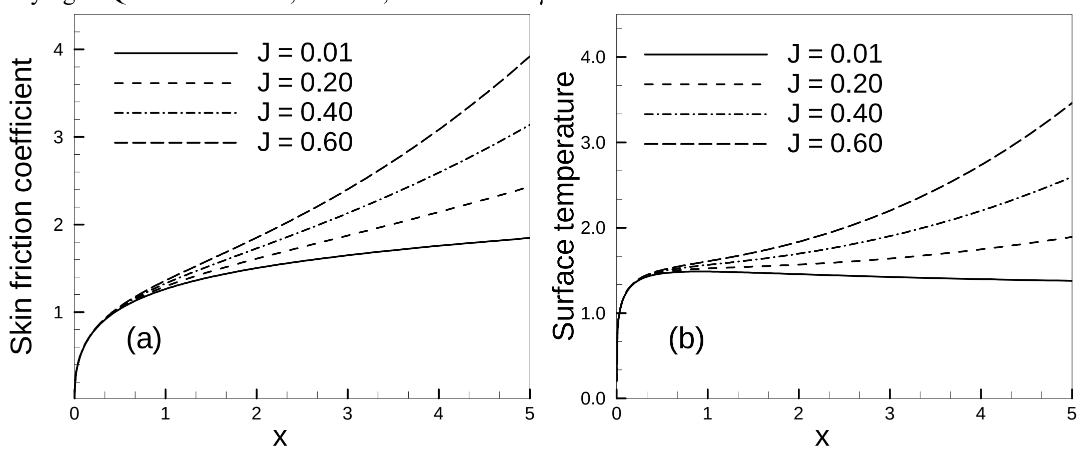

Figure 11(a) Local skin friction coefficient C_{fx} and (b) Surface temperature distribution θ(x, 0) against x for varying of J with Pr = 0.733, Q = 0.01, M = 0.01 and γ = 0.01.

5. CONCLUSION

The effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and joule heating have been studied numerically and graphically. From this investigation the following conclusions may be drawn

i) The velocity profile within the boundary layer increases for decreasing values of *M, Pr* and increasing values of *γ*, *Q* and *J*.

ii) The temperature profile within the boundary layer increases for increasing values of *M, γ, Q* and *J* and decreasing values of *Pr.*

iii) The local skin friction coefficient decreases for the increasing values of *M, Pr* and increases increasing values of *γ*, *Q* and *J.*

iv) An increase in the values of *M, γ, Q* and *J* leads to an increase in surface temperature distribution. Moreover, this decreases for increasing values of *Pr.*

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