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EFFECT OF TEMPERATURE DEPENDENT VISCOSITY INVERSELY PROPORTIONAL TO LINEAR FUNCTION OF TEMPERATURE ON MAGNETOHYDRODYNAMIC NATURAL CONVECTION FLOW ALONG A VERTICAL WAVY SURFACE

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ABSTRACT

In this paper, the effect of temperature dependent variable viscosity inversely proportional to linear function of temperature on Magnetohydrodynamic (MHD) natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. The governing boundary layer equations with associated boundary conditions for phenomenon are converted to non-dimensional form using suitable transformations. The resulting nonlinear system of partial differential equations are mapped into the domain of flat vertical plate and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. The solutions are obtained in terms of the skin friction coefficient, the rate of heat transfer, the streamlines and the isotherms over the whole boundary layer and are shown graphically for the effects of the pertinent parameters, such as the viscosity parameter (ε) and the magnetic parameter (M) for Prandtl number Pr = 0.73 and the amplitude of the wavy surface α = 0.3.

Key words: Magnetohydrodynamics, temperature dependent viscosity, natural convection, uniform surface temperature, Keller-Box method and wavy surface.

1. INTRODUCTION

It is necessary to study the heat transfer from an irregular surface because irregular surfaces are often present in many applications. It is often encountered in heat transfer devices to enhance heat transfer. Laminar natural convection flow from irregular surfaces can be used for transferring heat in several heat transfer devices, for examples, flat-plate solar collectors and flat-plate condensers in refrigerators and heat exchanger. One common example of a heat exchanger is the radiator used in car, in which the heat generated from engine transferred to air flowing through the radiator. Alam et al. [1] have studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. The viscosity of the fluid to be proportional to a linear function of temperature, two semi-empirical formulae were proposed by Charraudeau [2]. The effect of temperature dependent viscosity on natural convection heat transfer from a horizontal isothermal cylinder of elliptic cross section have been studied by Cheng [3]. Hossain et al. [4] investigated the natural convection flow past a permeable wedge for the fluid having temperature dependent viscosity and thermal conductivity. Molla et al. [6] studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Natural convection flow from an isothermal horizontal circular cylinder with temperature dependent viscosity was also investigated by Molla et al. [7]. Hossain et al. [8] investigated natural convection of a viscous fluid with viscosity inversely proportional to linear function of temperature from a vertical wavy cone. Numerical study on a vertical plate with variable viscosity and thermal conductivity has been investigated by Palani and Kim [9]. Nasrin et al. [10] investigated MHD free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature. The natural convection heat transfer from an isothermal vertical wavy surface was first studied by Yao [11, 12] and using an extended Prandtl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surface. Yet the preceding literature survey shows that in natural convection heat transfer the variation of viscosity with temperature and magnetic field along a vertical wavy surface has not been well investigated.

The present study is to incorporate the idea on the effects of temperature dependent viscosity inversely proportional to linear function of temperature in presence of strong magnetic field of electrically conducting fluid with free convection along a vertical wavy surface. However, it is known that viscosity may change significantly with temperature. The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique [5]. We have focused our attention on the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, the stream lines and the isotherms for selected values of parameters consisting of the magnetic parameter M and the viscosity variation parameter ε.

2. FORMULATION OF THE PROBLEM

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

where L is the characteristic length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Figure 1.

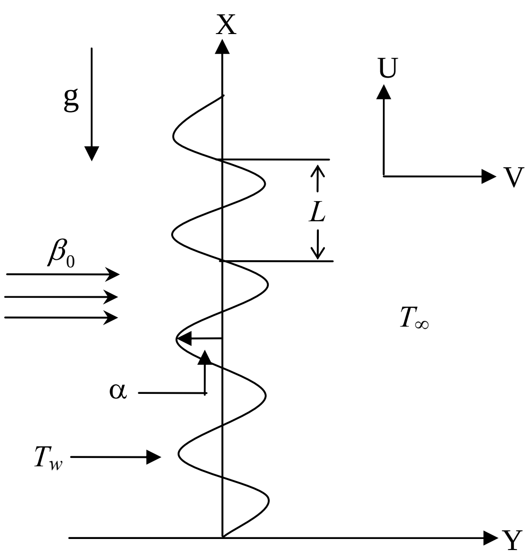

Figure 1. Physical model and coordinate system

Under the usual Boussinesq approximation, we consider the flow governed by the following boundary layer equations:

Continuity Equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

Momentum Equations

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{1}{\rho}\nabla.(\mu\nabla U) + g\beta(T - T_\infty) - \frac{\sigma_0\beta_0^2}{\rho}U \tag{3}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \frac{1}{\rho}\nabla.(\mu\nabla V) \tag{4}$$

Energy Equation

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{k}{\rho C_p}\nabla^2 T \tag{5}$$

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface and (U, V) are the velocity components parallel to (X, Y), $k(T)$ is the thermal conductivity and $\mu(T)$ is the dynamic viscosity of the fluid in the boundary layer region depending on the fluid temperature.

The boundary conditions for the present problem are

$$U = 0, V = 0, T = T_w \quad at\, Y = Y_w = \bar{\sigma}(X) \tag{6}$$

$$U = 0, \quad T = T_\infty, \quad P = p_\infty \quad as \quad Y \to \infty \tag{7}$$

There are very few forms of viscosity variation available in the literature. Among them we have considered that one which is appropriate for liquid introduced by Hossain et al. [8] as follows:

$$\mu = \frac{\mu_\infty}{1 + \varepsilon^*(T - T_\infty)} \tag{8}$$

where μ_∞ is the viscosity of the ambient fluid and ε^* is a constant evaluated at the film temperature of the flow $T_f = 1/2\,(T_w + T_\infty)$.

Following Yao [11], we now introduce the following non-dimensional variables

$$x = \frac{X}{L}, \quad y = \frac{Y - \bar{\sigma}}{L} Gr^{\frac{1}{4}}, \quad p = \frac{L^2}{\rho \nu^2} Gr^{-1} P$$

$$u = \frac{\rho L}{\mu_\infty} Gr^{-1/2} U, \quad v = \frac{\rho L}{\mu_\infty} Gr^{-1/4}(V - \sigma_x U),$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \sigma_x = \frac{d\bar{\sigma}}{dX} = \frac{d\sigma}{dx}, Gr = \frac{g\beta(T_w - T_\infty)}{\nu^2} L^3$$

Introducing the above dimensionless variables into Equations (2)–(5), the following dimensionless form of the governing equations are obtained:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x \frac{\partial p}{\partial y} + \frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2} - \frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - Mu + \theta \tag{10}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -Gr^{1/4}\frac{\partial p}{\partial y} + \frac{\sigma_x(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2} - \frac{\varepsilon\sigma_x(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - \sigma_{xx}u^2 \tag{11}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1+\sigma_x^2\right)\frac{\partial^2 \theta}{\partial y^2} \tag{12}$$

In the above equations, Pr, ε and M are, respectively known as the Prandtl number, the dimensionless viscosity parameter and dimensionless Magnetic parameter, which are defined as

$\Pr = \frac{C_p \mu_\infty}{k}$, $\varepsilon = \varepsilon^* (T_w - T_\infty)$ and $M = \frac{\sigma_0 \beta_0^2 L^2}{\mu Gr^{1/2}}$

Equation (11) indicates that the pressure gradient along the y-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along x-direction can be determined from the inviscid flow solution. Equation (11) further shows that $Gr^{1/4}\partial p/\partial y$ is O(1) and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from equations (10) and (11) leads to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2 - \frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} - \frac{M}{1+\sigma_x^2}u + \frac{1}{1+\sigma_x^2}\theta \tag{13}$$

The corresponding boundary conditions for the present problems then turn into

$$\left.\begin{array}{lll} u = v = 0, & \theta = 1 & at \quad y = 0 \\ u = \theta = 0, & p = 0 & as \quad y \to \infty \end{array}\right\} \tag{14}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta), \quad \eta = yx^{-1/4}, \quad \theta = \theta(x,\eta) \tag{15}$$

where η is the pseudo similarity variable and ψ is the stream function that satisfies the equation (9).

Introducing the transformations given in equation (15) into equations (13) and (12) the momentum and energy equations take the following forms,

$$\frac{(1+\sigma_x^2)}{(1+\varepsilon\theta)}f''' + \frac{3}{4}ff'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2 + \frac{1}{1+\sigma_x^2}\theta - \frac{Mx^{1/2}}{1+\sigma_x^2}f' - \frac{\varepsilon(1+\sigma_x^2)}{(1+\varepsilon\theta)^2}\theta' f'' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{16}$$

$$\frac{1}{\Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4}f\theta' = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \tag{17}$$

The boundary conditions (14) now take the following form:

$$\left.\begin{array}{ll} f(x,o) = f'(x,o) = 0, & \theta(x,o) = 1 \\ f'(x,\infty) = 0, & \theta(x,\infty) = 0 \end{array}\right\} \tag{18}$$

The rate of heat transfer in terms of the local Nusselt number, Nu_x and the local skin friction coefficient, C_{fx} take the following forms:

$$Nu_x (Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\,\theta'(x,o) \tag{19}$$

$$C_{fx}(Gr/x)^{1/4}/2 = \frac{\sqrt{1+\sigma_x^2}}{(1+\varepsilon)}f''(x,o) \tag{20}$$

3. METHOD OF SOLUTION

This paper concerns the natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface in presence of strong magnetic field and variable viscosity inversely proportional to linear function of temperature along a vertical wavy surface using the very efficient implicit finite difference method known as Keller box scheme developed by Keller [5]. This method has been extensively used recently by Hossain et al. [1, 4, 6, 7 and 8].

4. RESULT AND DISCUSSION

The governing equations (16) and (17) with the boundary conditions in equation (18) are solved numerically employing a very efficient implicit finite difference method together with Keller-box scheme. Numerical values of the shear stress in terms of the skin friction coefficients C_{fx}, the rate of heat transfer in terms of the Nusselt number Nu_x, the streamlines and the isotherms are presented graphically for different values of the viscosity parameter ε = 0.0 (constant viscosity) to 1.0, the magnetic parameter M

= 0.0 (non magnetic field) to 2.0 while Prandtl number Pr = 0.73 and the amplitude of the wavy surface α = 0.3.

The influence of the parameter ε = (0.0, 0.5 and 1.0) on the surface shear stress in terms of the local skin friction C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x are illustrated graphically in Figures 2(a) and 2(b) respectively when the values of amplitude of wavy surface α = 0.3, magnetic parameter M = 0.5 and Prandtl number Pr = 0.73. Figure 2(a) indicates that increasing the values of the viscosity-variation parameter ε, the skin friction coefficient decreases slowly along the downward direction of the plate. On the other hand it can be shown from Figure 2(b) that the value of the rate of heat transfer along the wavy surface increases for increasing the values of the viscosity-variation parameter ε. Here we conclude that for high viscous fluid inversely proportional function of temperature the skin-friction coefficient is slow and the corresponding rate of heat transfer is large. In Figure 2(a), the maximum values of local skin friction coefficient C_{fx} are 0.86640, 0.78213 and 0.77294 for ε = 0.0, 0.5 and 1.0 respectively which occur at different values of x and it is seen that the local skin friction coefficient C_{fx} decreases by 10.79% as ε increases from 0.0 to 1.0. Again Figure 2(b) shows that the rate of heat transfer increases 7.82% due to the increased value of ε. Increasing values of ε lead to increase the amplitude of the Nu_x.

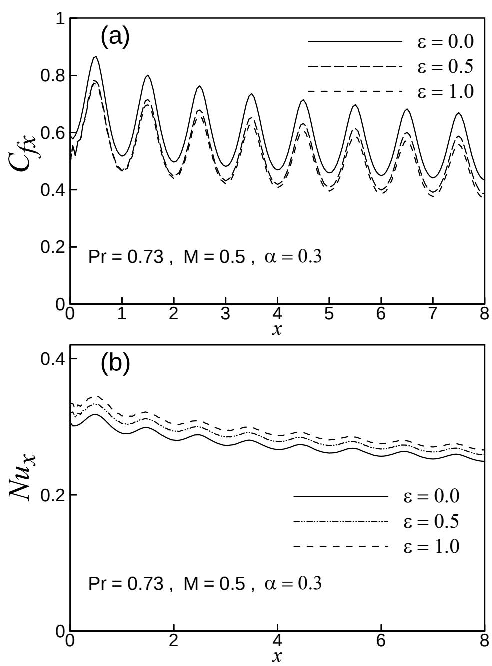

Figure 2. Variation of (a) skin-friction coefficient (b) rate of heat transfer against x for varying of ε with M = 0.5, α = 0.3 and Pr = 0.73.

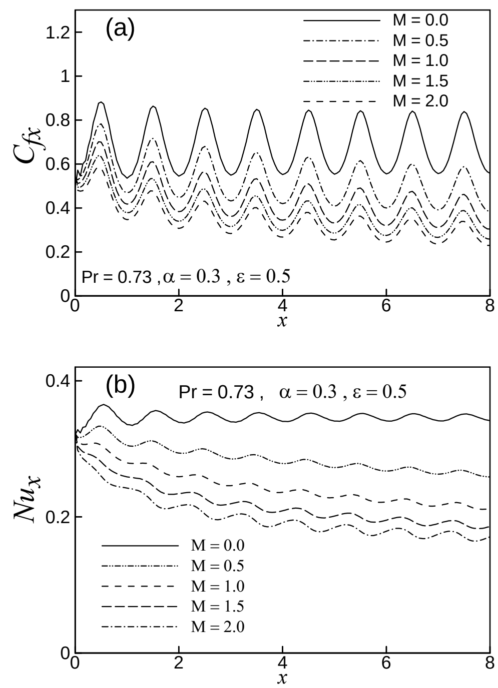

Figure 3. Variation of (a) skin-friction coefficient (b) rate of heat transfer against x for varying of M with ε = 0.5, α = 0.3 and Pr = 0.73.

The effect of the magnetic parameter M = (0.0, 0.5, 1.0, 1.5, 2.0) on the surface shear stress in terms of the local skin friction C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x are depicted graphically in Figures 3(a) and 3(b) respectively when the values of amplitude of wavy surface α = 0.3, viscosity parameter ε = 0.5, and Prandtl number Pr = 0.73. As electrically conducting fluid in presence of magnetic field generates electrical current, the magnetic field is changed and the fluid motion is moderated. As a result, the velocity gradient $f''(x, 0)$ decreases with the increase of the magnetic parameter M. The same result is observed on the local rate of heat transfer Nu_x due to that an increase in the magnetic parameter M at different position of x. The skin friction coefficient and the rate of heat transfer coefficient decrease by 33.55% and 11.71% respectively as M increases from 0.0 to 2.0.

Figures 4 and 5 illustrate the effect of the temperature dependent viscosity variation parameter ε on the development of streamlines and isotherms respectively which are plotted for Prandtl number Pr = 0.73, amplitude of wavy surface α = 0.3 and M = 0.5. We find that for ε = 0.0 the value of ψ_{max} = 6.46, for ε = 0.5 the value of ψ_{max} = 6.68 and for ε = 1.0 the value of ψ_{max} = 6.78. From Figure 4, it is seen that the effect of viscosity parameter ε, the flow rate in the boundary layer increases. From Figure 5, we observe that owing to the effect of ε, the thermal state of the fluid decreases causing the thermal boundary layer decrease.

The effect of variation of the surface roughness on the streamlines and isotherms for the values of M equal to 0.0, 1.0 and 2.0 are depicted by the Figure 6 and Figure 7 respectively while Prandtl number Pr = 0.73, amplitude of wavy surface α = 0.3 and viscosity variation parameter ε = 0.5. We observe from Figure 7 that as the values of magnetic parameter M increases the thermal boundary layer thickness becomes higher gradually. Figure 6 depicts that the maximum values of ψ decreases steadily while the values of M increases. The maximum values of ψ, that is, ψ_{max} are 9.37, 4.69 and 3.05 for magnetic parameter M = 0.0, 1.0 and 2.0 respectively. Finally we conclude that for much roughness of the surface with the effect of magnetic parameter the velocity of the fluid flow decreases in the boundary layer.

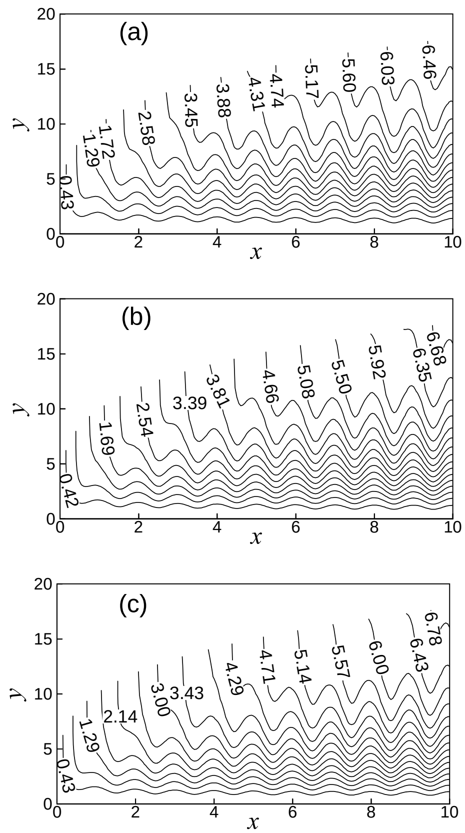

Figure 4. Streamlines for (a) ε = 0.0, (b) ε = 0.5 and (c) ε = 1.0 while the magnetic parameter M = 0.5, amplitude of wavy surface α = 0.3 and Prandtl number Pr = 0.73.

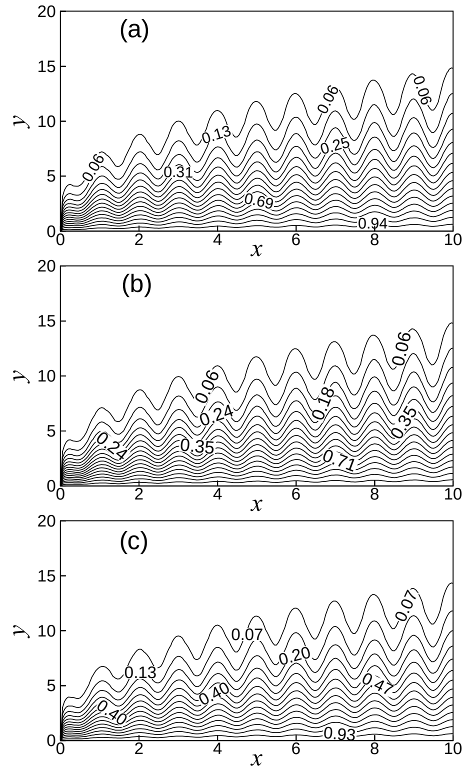

Figure 5. Isotherms for (a) ε = 0.0, (b) ε = 0.5 and (c) ε = 1.0while M = 0.5, α = 0.3 and Pr = 0.73.

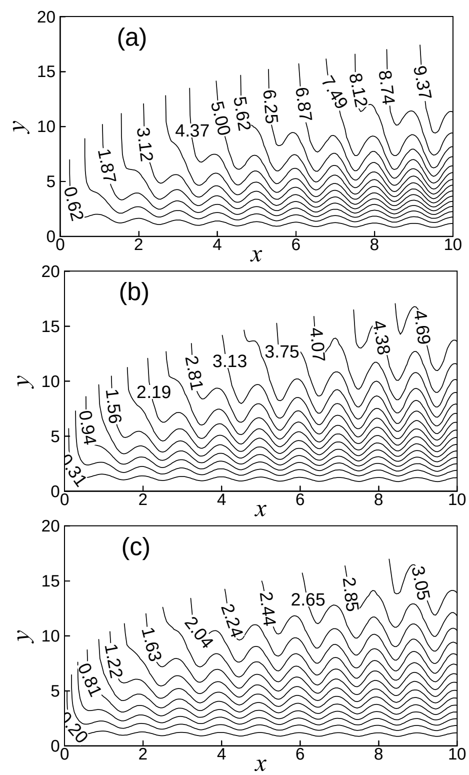

Figure 6. Streamlines for (a) M = 0.0, (b) M = 1.0 and (c) M = 2.0 while ε = 0.5, α = 0.3 and Pr = 0.73.

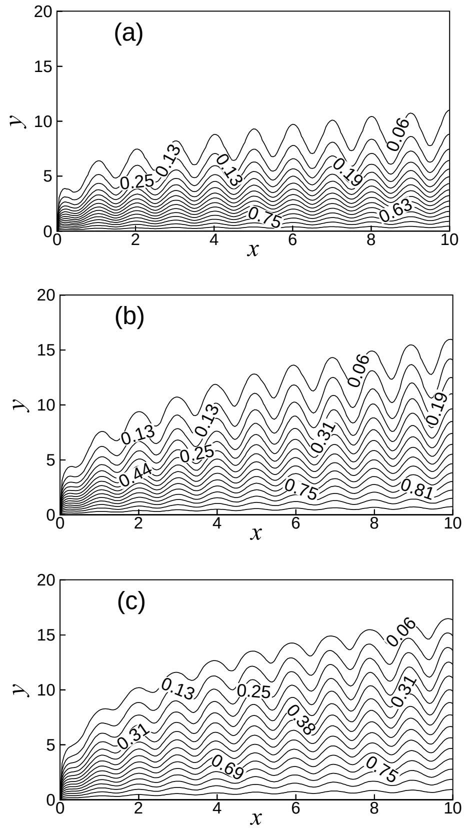

Figure 7. Isotherms for (a) M = 0.0, (b) M = 1.0 and (c) M = 2.0 while ε = 0.5, α = 0.3 and Pr = 0.73.

5. CONCLUSION

The effect of temperature dependent variable viscosity inversely proportional to linear function of temperature on Magnetohydrodynamic (MHD) natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. From the present investigation the following conclusions may be drawn:

- The effect of increasing viscosity parameter ε results in decreasing the local skin friction coefficient C_{fx} and increasing the local rate of heat transfer Nu_x.
- An increase in the values of M leads to decrease the skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x.
- The flow rate decreases and the thermal boundary layer increases when the effect of magnetic field is considered.
- The streamlines increase and the thermal boundary layer decrease when viscosity parameter ε increases.

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