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EFFECTS OF TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY AND VISCOUS DISSIPATION ON CONJUGATE FREE CONVECTION FLOW ALONG A VERTICAL FLAT PLATE

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ABSTRACT

The effect of the temperature dependent thermal conductivity (TDTC) on conjugate free convection flow along a vertical flat plate with viscous dissipation has been investigated. The transformed non-linear ordinary partial differential equations are solved using the implicit finite difference method with Keller–box scheme. Numerical results for the velocity and temperature profiles as well as local skin friction co-efficient and surface temperature distributions for different values of thermal conductivity variation parameter, viscous dissipation parameter and Prandtl number are presented graphically. The values of the Prandtl number are considered to be 0.733, 1.099, 1.63 and 2.18 that corresponds to air, water, glycerin and sulfur dioxide, respectively.

Keywords: *TDTC, Conjugate free convection, vertical flat plate, viscous dissipation, finite difference method.*

1. INTRODUCTION

Free convection heat transfer and fluid flow in enclosures with various form and wall conductions has been studied widely in current year due to its wide range applications, such as building thermal design, solar energy collector etc. Aydin [1] studied conjugate heat transfer through a double pane window. On the other hand the thermal interaction between laminar film condensation forced convection along a conducting wall was investigated by Chen and Chang [2]. The axial heat conduction effect in a vertical flat plate on a free convection heat transfer was studied by Miyamoto *et al.* [3]. Pozzi and Lupo [4] investigated the coupling of conduction with laminar natural convection along a flat plate. Merkin and Pop [5] presented conjugate free convection on a vertical surface. In all the aforementioned analyses the effects of temperature dependent thermal conductivity has not been considered. But, laminar free convection flow from an isothermal sphere immerged in a fluid with thermal conductivity proportional to linear function of temperature has been studied by Molla *et al* [6]. Gebeart [7] has shown that the viscous dissipation effect plays an important role in natural convection for various devices which are subjected to large deceleration or which operate at high rotative speeds and also in strong gravitational field processes on large scales (on large planets) and geological processes. Takhar and Soundalgekar [8] have studied dissipation effects on MHD free convection flow past semi-infinite vertical plate. Therefore the objective of the present work is to investigate the effect of (TDTC) on the free convection flow along a vertical flat plate with heat conduction and viscous dissipation.

2. FORMULATION OF THE PROBLEM

Let us consider a steady two-dimensional natural convection flow of viscous and incompressible fluid along a vertical flat plate of length l and thickness b (See Fig.1).It is assumed that the temperature at the outside surface of the plate is maintained at a constant temperature T_b, where $T_b > T_\infty$, the ambient temperature of the fluid.

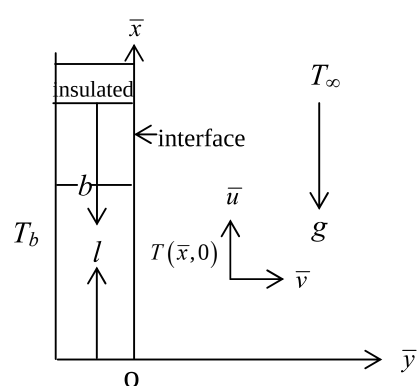

Fig.1: Physical model and coordinate

The *x*-axis is taken along the vertical flat plate in the upward direction and the *y*-axis normal to the plate.

The governing equations of such flow under the usual boundary layer and the Boussinesq approximations with temperature dependent thermal conductivity are given below:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T_f - T_\infty) \quad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho c_p}\frac{\partial}{\partial \bar{y}}(\kappa_f \frac{\partial T_f}{\partial \bar{y}}) + \frac{\upsilon}{c_p}\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 \quad (3)$$

Here we will consider the form of the temperature dependent thermal conductivity, which is proposed by Charraudeau [9]

$$\kappa_f = \kappa_\infty[1 + \delta(T_f - T_\infty)] \quad (4)$$

where κ_∞ is the thermal conductivity of the ambient fluid and δ is a constant.

The appropriate boundary conditions to be satisfied by the above equations are

$$\left.\begin{aligned} &\bar{u} = 0, \qquad \bar{v} = 0 \\ &T_f = T(\bar{x},0), \frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_s}{b\kappa_f}(T_f - T_b) \end{aligned}\right\}, at\,\bar{y} = 0,\ \bar{x} > 0 \quad (5)$$

$$\bar{u} \to 0,\ T_f \to T_\infty \quad as \quad \bar{y} \to \infty, \quad \bar{x} > 0$$

The non-dimensional governing equations and boundary conditions can be obtained from equations (1) - (5) using the following non-dimensional quantities

$$x = \frac{\bar{x}}{l}, y = \frac{\bar{y}}{l}Gr^{\frac{1}{4}}, u = \frac{\bar{u}l}{\upsilon}Gr^{-\frac{1}{2}}, v = \frac{\bar{v}l}{\upsilon}Gr^{-\frac{1}{4}}, \theta = \frac{T_f - T_\infty}{T_b - T_\infty}, \quad (6)$$

$$Gr = \frac{g\beta l^3(T_b - T_\infty)}{\upsilon^2}$$

where l is the length of the plate, Gr is the Grashof number, θ is the non-dimensional temperature.

By applying the dimensionless quantities defined by equation (6) in to equations (1) to (3) we get the following non -dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \quad (8)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2} + \frac{\gamma}{\Pr}\left(\frac{\partial \theta}{\partial y}\right)^2 + N\left(\frac{\partial u}{\partial y}\right)^2 \quad (9)$$

where $Pr = \left(\mu\, c_p\right)/\,k_\infty$ is the Prandtl number,

$\gamma = \delta(T_b - T_\infty)$, is the dimensionless thermal conductivity variation parameter and

$N = \nu^2 Gr / l^2 c_p (T_b - T_\infty)$ is the dimensionless viscous dissipation parameter.

The corresponding boundary conditions (5) take the following form

$$u = 0,\ v = 0,\ \ \theta - 1 = (1+\gamma\theta)\ p\frac{\partial \theta}{\partial y}, at\,y = 0,\ x > 0 \quad (10)$$

$$u \to 0, \theta \to 0 \quad as\ y \to \infty, x > 0$$

where $p = \left(k_\infty\, b\right)/\left(k_s l\right) Gr^{1/4}$ is the conjugate conduction parameter. This coupling parameter determines the significance of the conduction resistance within the wall. In the present investigation we have considered p = 1.

To solve the equations (8) and (9) subject to the boundary conditions (10) the following transformations are then introduced

$$\psi = x^{\frac{4}{5}}(1+x)^{\frac{1}{-20}}f(x,\eta),\ \eta = y\,x^{-\frac{1}{5}}(1+x)^{-\frac{1}{20}}, \quad (11)$$

$$\theta = x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h(x,\eta)$$

here η is the similarity variable and ψ is the non-dimensional stream function which satisfies the continuity equation and is related to the velocity components in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Moreover $h\ (x, \eta)$ represents the dimensionless temperature. The momentum and energy equations are transformed for the new co-ordinate system. Thus we get

$$f''' + \frac{16 + 15x}{20(1+x)} f\,f'' - \frac{6 + 5x}{10(1+x)} f'^2 + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \quad (12)$$

$$\frac{1}{\Pr}h'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}} h\,h'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}} h'^2 + \frac{16 + 15x}{20(1+x)} f\,h' - \frac{1}{5(1+x)} f'\,h + N\,x\,f''^2 = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \quad (13)$$

where prime denotes partial differentiation with respect to η. The boundary conditions as mentioned in equation (10) then take the following form

$$\left.\begin{aligned} &f(x,0) = f'(x,0) = 0, \\ &h'(x,0) = \frac{x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h\ (x,0) - 1}{(1+x)^{-\frac{1}{4}} + \gamma\,x^{\frac{1}{5}}(1+x)^{-\frac{9}{20}}h(x,0)} \\ &f'(x,\infty) \to 0,\ h\ (x,\infty) \to 0 \end{aligned}\right\} \quad (14)$$

From the process of numerical computation, in practical point of view, it is important to calculate the values of the surface shear stress in terms of the skin friction coefficient. This can be written in the non-dimensional form as

$$C_f = \left(Gr^{-3/4}\,l^2\right)/\left(\mu\nu\right)\tau_w \quad (15)$$

where $\tau_w[= \mu(\partial\bar{u}/\partial\bar{y})_{\bar{y}=0}]$ is the shearing stress. Using the new variables described in equation (6), the local skin friction co-efficient can be written as

$$C_{fx} = x^{2/5}(1+x)^{-3/20}f''(x,0) \quad (16)$$

The numerical values of the surface temperature distribution are obtained form the relation

$$\theta(x,0) = x^{1/5}(1+x)^{-1/5}\,h(x,0) \tag{17}$$

3. COMPARISON WITH PREVIOUS WORK AND PROGRAMME VALIDATION

A comparison of the surface temperature and local skin friction coefficient obtained in the present work with $x^{\frac{1}{5}} = \xi$ and $\gamma = 0$ and obtained by Merkin and Pop (1996) and Pozzi and Lupo (1988) have been shown in Table-1 and Table-2, respectively. It is clearly seen that there is an excellent agreement among the respective results.

Table-1: Comparison of the present numerical results of surface temperature with Prandtl number Pr = 0.733 and γ = 0

$\theta(x,0)$			
$x^{\frac{1}{5}} = \xi$	Pozzi and Lupo (1988)	Merkin and Pop (1996)	Present work
0.7	0.651	0.651	0.651
0.8	0.684	0.686	0.687
0.9	0.708	0.715	0.716
1.0	0.717	0.741	0.741
1.1	0.699	0.762	0.763
1.2	0.640	0.781	0.781

Table-2: Comparison of the present numerical results of local skin friction coefficient with Prandtl number Pr = 0.733 and γ = 0

C_{fx}			
$x^{\frac{1}{5}} = \xi$	Pozzi and Lupo (1988)	Merkin and Pop (1996)	Present work
0.7	0.430	0.430	0.424
0.8	0.530	0.530	0.529
0.9	0.635	0.635	0.635
1.0	0.741	0.745	0.744
1.1	0.829	0.859	0.860
1.2	0.817	0.972	0.975

4. RESULTS AND DISCUSSION

The present work is used to analyze the effect of thermal conductivity variation due to temperature on free convection flow along a vertical flat plate in presence of heat conduction and viscous dissipation. The values of the Prandtl number are considered to be 0.733, 1.099, 1.63 and 2.18 that corresponds to air, water, glycerin and sulfur dioxide respectively.

The effect of thermal conductivity variation parameter γ on the velocity and the temperature profiles within the boundary layer with N = 0.01 and Pr = 0.733 are shown in Fig. 2 and Fig. 3, respectively. It is seen from Fig. 2 and Fig. 3 that the velocity and temperature increase within the boundary layer with the increasing value of γ. It means that the velocity boundary layer and the thermal boundary layer thickness increase for increasing values of γ.

The effect of viscous dissipation parameter on the velocity and temperature within the boundary layer with γ = 0.01 and Pr = 0.733 are shown in Fig. 4 and Fig. 5, respectively. It is seen that from Fig. 4 and Fig. 5 that the velocity and temperature increase within the boundary layer with the increasing value of N.

Fig. 6 and Fig. 7 illustrate the velocity and temperature profiles for different values of Prandtl number Pr with N = 0.01 and γ = 0.01. From Fig. 6, it can be observed that the velocity decreases as well as its position moves toward the interface with the increasing Pr. From Fig. 7, it is seen that the temperature profiles shift downward with the increasing values of Pr. Since the viscosity is the resistance to flow of a fluid, so the velocity decreases for increasing value of Pr.

Fig. 8 and Fig. 9 illustrate the effect of the thermal conductivity variation parameter on the local skin friction coefficient and surface temperature distribution against x with N = 0.01 and Pr = 0.733. It is also seen that the local skin friction coefficient increases for the increasing γ. From Fig. 8, it is seen that the surface temperature increases for the increasing γ. This is to be expected because the higher value for the thermal conductivity variation parameter accelerates the fluid flow and increases the temperature as mentioned in Fig. 2 and Fig. 3, respectively.

The effect of viscous dissipation parameter on the local skin friction coefficient and surface temperature distribution against x with γ = 0.01 and Pr = 0.733 are shown in Fig. 10 and Fig. 11, respectively. It is observed from Fig. 10 that the local skin friction coefficient increases for the increasing values of N. From Fig. 11 it can be seen that the surface temperature distribution increase for increasing values of N.

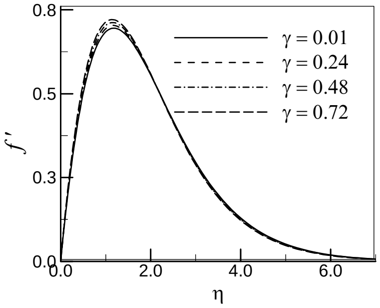

Fig. 2: Variation of velocity profiles for different values of γ with $N = 0.01$ and $Pr = 0.733$

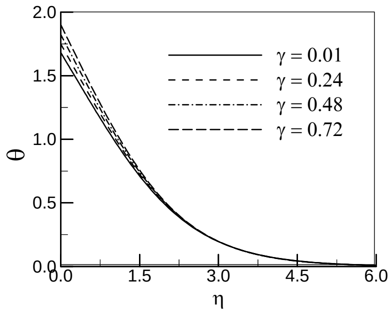

Fig. 3: Variation of temperature profiles for different values of γ with $N = 0.01$ and $Pr = 0.733$

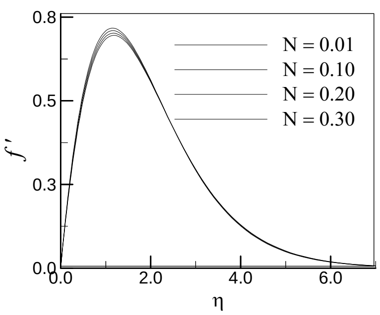

Fig. 4: Variation of velocity profiles for different values of N with $\gamma = 0.01$ and $Pr = 0.733$

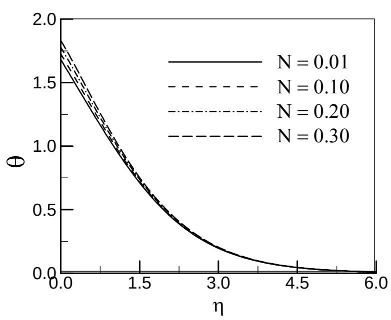

Fig. 5: Variation of temperature profiles for different values of N with $\gamma = 0.01$ and $Pr = 0.733$

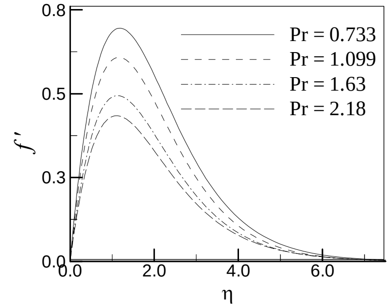

Fig. 6: Variation of velocity profiles for different values of Pr with $N = 0.01$ and $\gamma = 0.01$

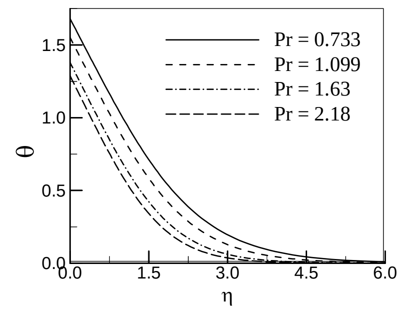

Fig. 7: Variation of temperature profiles for different values of Pr with $N = 0.01$ and $\gamma = 0.01$

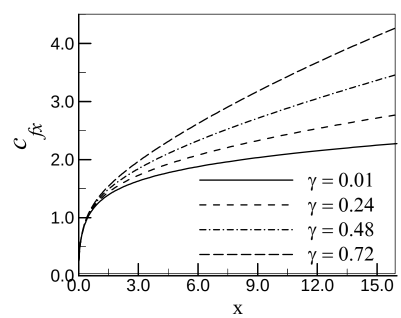

Fig.8: Variation of skin friction for different values of γ with $N = 0.01$ and $Pr = 0.733$

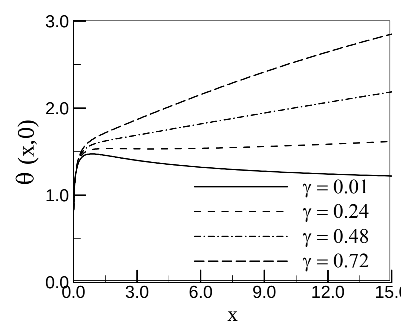

Fig. 9: Variation of surface temperature for different values of γ with $N = 0.01$ and $Pr = 0.733$

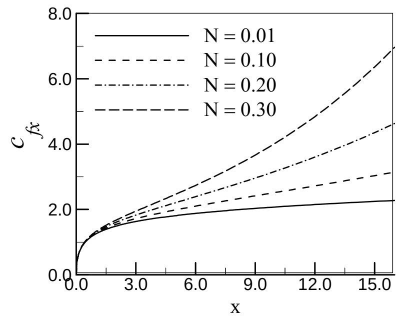

Fig. 10: Variation of skin friction for different values of N with γ = 0.01 and Pr = 0.733

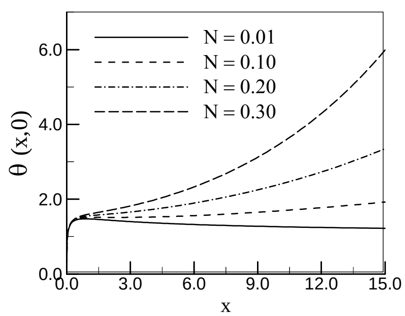

Fig. 11: Variation of surface temperature for different values of N with γ = 0.01 and Pr = 0.733

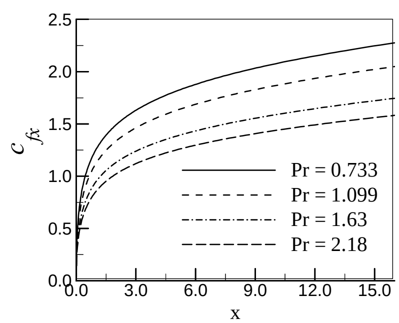

Fig. 12: Variation of skin friction for different values of Pr with N = 0.01 and γ = 0.01

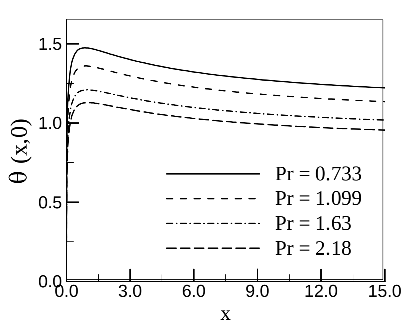

Fig. 13: Variation of surface temperature for different values of Pr with N = 0.01 and γ = 0.01

Fig. 12 and Fig. 13 deal with the effect of Prandtl number on the local skin friction coefficient and surface temperature distribution against x with N = 0.01 and γ = 0.01. It can be observed from Fig. 12 that the local skin friction coefficient decreases for the increasing Pr. From Fig. 13. It can be seen that the surface temperature distribution decreases for the increasing values of Pr. Since the skin friction decreases for increasing value of Pr as a result the temperature distribution within the boundary layer decreases for increasing Pr and also surface temperature decreases for increasing Pr.

5. CONCLUSIONS

In this paper the effect of thermal conductivity variation due to temperature and conduction on free convection flow along a vertical flat plate with viscous dissipation has been studied. From the present numerical investigation the following conclusions may be drawn:

- The velocity within the boundary layer increases for decreasing values of Prandtl number and increasing values of the thermal conductivity variation and viscous dissipation parameters.
- The temperature within the boundary layer increases for the increasing values of thermal conductivity variation parameter and viscous dissipation parameter and decreasing values of the Prandtl number.
- The local skin friction co-efficient decreases for the increasing values of Prandtl number and decreasing values of the thermal conductivity variation and viscous dissipation parameters.
- An increase in the values of the thermal conductivity variation parameter, viscous dissipation parameter leads to an increase in the surface temperature and decreases for the increasing values of Prandtl number.

NOMENCLATURE

b	Plate thickness
C_{fx}	Local skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
g	Acceleration due to gravity
Gr	Grashof number
l	Length of the plate
N	Dimensionless viscous dissipation parameter
P	Conjugate conduction parameter
Pr	Prandtl number
T	Temperature of the interface
T_b	Temperature at outside surface of the plate
T_f	Temperature of the fluid
T_∞	Temperature of the ambient fluid
$\bar{u}$	Velocity component in x- direction
$\bar{v}$	Velocity component in y- direction

u Dimensionless velocity component in x-direction
v Dimensionless velocity component in y-direction
$\overline{x}$ Cartesian co-ordinate along the vertical flat plat in the upward direction
$\overline{y}$ Cartesian co-ordinate normal to the plate
x Dimensionless Cartesian co-ordinate along the vertical flat plat in the upward direction
y Dimensionless Cartesian co-ordinate normal to the plate

Greek Symbols

β Co-efficient of thermal expansion
γ Thermal conductivity variation parameter
∇ Vector differential operator
η Similarity variable
θ Dimensionless temperature
$\theta(x,0)$ Surface temperature distribution
κ_∞ Thermal conductivity of the ambient fluid
κ_s Thermal conductivity of the solid
κ_f Thermal conductivity of the fluid
μ Viscosity of the fluid
υ Kinematic viscosity
ρ Density of the fluid inside the boundary layer
τ_w Dimensionless shearing stress
ψ Dimensionless stream function

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