Proceedings of MARTEC 2010
The International Conference on Marine Technology
11-12 December 2010, BUET, Dhaka, Bangladesh

MHD-CONJUGATE FREE CONVECTION FLOW FROM AN ISOTHERMAL HORIZONTAL CYLINDER WITH STRESS WORK

NHM. A. Azim[1], ATM. M. Rahman[2] and M. K. Chowdhury[3]

[1]School of Business Studies, Southeast University, Dhaka, Bangladesh,
Email: nhmarif1@gmail.com

[2]Department of Computer Science and Engineering,, Dhaka International University, Dhaka, Bangladesh
Email: atmmrahman2005@gmail.com

[3]Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh
Email: mkamal@math.buet.ac.bd

ABSTRACT

The effects of conduction and stress work on Magnetohydrodynamic(MHD) free convection flow from an isothermal horizontal cylinder along the outer surface from the lower stagnation point to the upper stagnation point are investigated. The developed governing equations and the associated boundary conditions are made dimensionless using a suitable transformation then the non-dimensional governing equations are solved using the implicit finite difference method with Keller box-scheme. Numerical outcomes are found for different values of the magnetic parameter, stress work parameter and conjugate conduction parameter. Results for the details of the velocity profiles and the temperature distributions as well as the skin friction coefficients and the rate of heat transfer are shown graphically and discussed.

Keywords: *Free convection, Conduction, MHD, Stress work, Horizontal cylinder, Finite difference method.*

1. INTRODUCTION

Many researchers investigated natural convection flow from a horizontal cylinder [1-4] under diverse surface boundary conditions (isothermal, uniform heat flux and mixed boundary conditions) using different mathematical technique. The conjugate heat transfer process formed by the interaction between the conduction inside the solid and the convection flow along the solid surface has a significant importance in many practical applications. Gdalevich and Fertman [5] studied the conjugate problems of natural convection. Miyamoto et al. [6] analysed the effects of axial heat conduction in a vertical flat plate on free convection heat transfer. Pozzi et al. [7] investigated the entire thermo-fluid-dynamic (TFD) field resulting from the coupling of natural convection along and conduction inside a heated flat plate by means of two expansions, regular series and asymptotic expansions. Moreover, Kimura and Pop [8] analysed conjugate natural convection from a horizontal circular cylinder.

MHD flow and heat transfer process are now an important research area due to its potential application in engineering and industrial fields. A considerable amount of research has been done in this field. Wilks et al. [9] studied MHD free convection about a semi-infinite vertical plate in a strong cross field. Aldoss et al. [10] analysed MHD mixed convection from a horizontal circular cylinder. El-Amin [11] found out the combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal circular cylinder embedded in a fluid saturated porous medium. The influence and importance of viscous dissipation and stress work effects in laminar flows have been examined by Gebhart [12]. Later the numerical solution of the effect of viscous dissipation and pressure stress work in natural convection along a vertical isothermal plate studied by Pantokratoras [13] without any approximation.

The objective of the present paper is to obtain the numerical result of MHD-conjugate free convection flow from an isothermal horizontal circular cylinder considering stress work effect with a complete discussion.

2. MATHEMATICAL ANALYSIS

Let us consider a steady natural convection flow of a viscous incompressible and electrically conducting fluid from an isothermal horizontal circular cylinder of radius a placed in a fluid of uniform temperature T_∞. The cylinder has a heated inner region of temperature T_b and the thickness of the circular cylinder is b with $T_b > T_\infty$. A uniform magnetic field having strength B_0 is acting normal to the cylinder surface. The $\bar{x}$-axis is taken along the circumference of the cylinder measured from the lower stagnation point and the $\bar{y}$-axis is taken normal to the surface. It is assumed the fluid properties to be constant and the induced magnetic field is ignored. The effects of stress work in the flow region and conduction from inner surface to the outer surface considered in the present study.

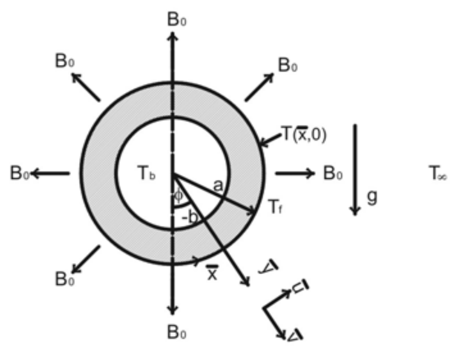

Fig. 1: Physical Model and coordinate system

Under the balance laws of mass, momentum and energy and with the help of Boussinesq approximation for the body force term in the momentum equation, the equations governing this boundary-layer natural convection flow can be written as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(T_f - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B_0^{\,2}\bar{u}}{\rho} \qquad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T_f}{\partial \bar{y}^2} + \frac{T\beta \bar{u}}{\rho c_p}\frac{\partial \bar{p}}{\partial \bar{x}} \qquad (3)$$

The physical situation of the system suggests the following boundary conditions

$$\left.\begin{aligned} &\bar{u} = \bar{v} = 0,\ T_f = T(\bar{x},0) \\ &\partial T_f / \partial \bar{y} = \kappa_s\left(T_f - T_b\right)/ b\kappa_f \end{aligned}\right\} on\ \bar{y} = 0,\ x > 0 \qquad (4)$$

$$\bar{u} \to 0,\ T_f \to T_\infty\ as\ \bar{y} \to \infty,\ \bar{x} > 0$$

The governing equations and the boundary conditions (1)-(4) can be made non-dimensional, using the Grashof number $Gr = \left(g\beta a^3 (T_b - T_\infty)\right)/\nu^2$ which is assumed large and the following non-dimensional variables:

$$x = \frac{\bar{x}}{a},\ y = \frac{\bar{y}}{a}Gr^{\frac{1}{4}},\ u = \frac{\bar{u}a}{\nu}Gr^{-\frac{1}{2}}$$

$$v = \frac{\bar{v}a}{\nu}Gr^{-\frac{1}{4}},\ \theta = \frac{T_f - T_\infty}{T_b - T_\infty}$$

Where θ is the dimensionless temperature. The non dimensional forms of the equations (1)-(3) are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \sin x \qquad (6)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + -\varepsilon\left(\frac{T_\infty}{T_b - T_\infty}u + u\theta\right) \qquad (7)$$

Where $M = (\sigma a^2 B_0^{\,2})/(\nu \rho Gr^{1/2})$ is the magnetic parameter, $\varepsilon = g\beta a / c_p$ is the stress work parameter and $Pr = \mu c_p / \kappa$ is the Prandtl number.

The boundary conditions (4) can be written as in the following dimensionless forms:

$$\left. u = v = 0,\ \theta - 1 = p\frac{\partial \theta}{\partial y}\right\} on\ \ y = 0,\ x > 0 \qquad (8)$$

$$u \to 0,\ \theta \to 0\ as\ y \to \infty,\ x > 0$$

Where $p = \left(b\kappa_f Gr^{1/4}\right)/\left(a\kappa_s\right)$ is the conjugate conduction parameter. The present problem is governed by the magnitude of p. The values of $\boldsymbol{p}$ depends on b/a, k_f/k_s and Gr. The ratios b/a and k_f/k_s are less than one where as Gr is large for free convection. Therefore the value of p is zero (b=0) or greater than zero.

To solve equation (5)-(7), subject to the boundary condition (8), we assume following transformation:

$$\psi = x f\left(x, y\right) \qquad (9)$$

Where ψ is the stream function usually defined as

$$u = \partial\psi / \partial y,\ v = -\partial\psi / \partial x \qquad (10)$$

Substituting (10) into the equations (5)-(7), the new forms of the dimensionless equations (6) and (7) are

$$f''' + ff'' - f'^2 - Mf' + \theta\frac{\sin x}{x} = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \qquad (11)$$

$$\frac{1}{\Pr}\theta'' + f\theta' - \varepsilon\left(x f'\frac{T_\infty}{T_b - T_\infty} - xf'\theta\right) = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \qquad (12)$$

In the above equations primes denote differentiation with respect to y. The corresponding boundary conditions (8) take the following form

$$f = f' = 0,\ \theta - 1 = p\frac{\partial \theta}{\partial y}\ \ at\ \ y = 0,\ x > 0 \qquad (13)$$

$$f' \to 0,\ \theta \to 0\ as\ y \to \infty,\ x > 0$$

Principle physical quantities, the shearing stress and the rate of heat transfer in terms of skin friction coefficient C_f and Nusselt number Nu respectively can be written as:

$$C_f\, Gr^{1/4} = x\, f''(x,0) \qquad (14)$$

$$Nu\, Gr^{-1/4} = -\theta(x,0) \qquad (15)$$

The results of the velocity profiles and temperature distributions can be calculated by the following relations respectively.

$$u = f'(x,y)\ ,\ \theta = \theta(x,y) \qquad (16)$$

3. METHOD OF SOLUTION

Equation (11) and (12) are solved numerically based on the boundary conditions as described in equation (13) using one of the most efficient and accurate methods known as implicit finite difference method with Keller box scheme [15, 16].

4. RESULTS AND DISCUSSION

The conjugate heat transfer analysis from an isothermal horizontal circular cylinder considering stress work effect is the main purpose of the present work. The Prandtl number is considered 1.0 for the simulation which corresponds to steam.

A comparison of the local Nusselt number and the local skin friction factor obtained in the present work with M = 0.0, ε = 0.0, p = 0.0 and Pr = 1.0 and obtained by Merkin [1] and Nazar et al. [14] have been shown in Tables 1 and 2 respectively. There is an excellent agreement among these three results.

Figs.2 and 3 illustrate the velocity and temperature distribution against y for different values of the magnetic parameter and the skin friction coefficient and the heat transfer rate against x for varying magnetic parameter with p =1.0 and ε =0.01 are depicted in figs.4 and 5 respectively. The magnetic field opposes the fluid flow as a result the peak velocity decreases with the increasing M as shown in fig. 2. Consequently, the separation of the boundary layer occurs earlier and the momentum boundary layer becomes thicker. From Fig. 3 it can be observed that the increasing value magnetic parameter increases temperature within the boundary layer, this is expected as there is an interaction with magnetic field with fluid flow. Thus, the magnetic parameter increases the thickness of the thermal boundary layer. Temperature at the interface also varies with different M since the conduction is considered within cylinder.

The Magnetic force opposes the flow, as mentioned earlier, and reduces the shear stress at the wall as illustrated in Fig. 4. Moreover, the heat transfer rate also decreases as temperature difference between solid surface and boundary layer region is reduced as revealed in Fig. 5.

The velocity profiles, temperature distributions, local skin friction coefficients and the heat transfer rate for different values of stress work parameter ε are presented in Fig. 6, Fig. 7, Fig. 8 and Fig. 9, respectively with p =1.0 and M =0.1. Increasing value of the stress work parameter containing gravitational force ***g*** work against the buoyancy force as a result the motion of the fluid motion is decreased as plotted in Fig.6. The reduced velocity decelerates fluid flow which ultimately decreases the shear stress at the wall which is observed from fig.8.

On the other hand from fig.7 it could be concluded that the temperature within the boundary layer decreases for increasing stress work parameter. The decreased temperature for increasing stress work parameter within the boundary layer reduced the temperature difference between the boundary layer region and the core region eventually increases heat transfer rate as illustrated in Fig.9.

The velocity profiles and temperature distributions for different values of conjugate conduction parameter p are presented in fig.10 and fig.11, respectively with M = 0.1 and ε = 0.01. It is observed that both the velocity profile and temperature distribution decrease for increasing p. It is expected because increase value of conjugate conduction resists heat conduction from the solid to the boundary layer.

Fig.12 and fig.13 depict the skin friction coefficient and the heat transfer rate for different values of conjugate conduction parameter p, respectively. It can be noted from these two figures that, both the skin friction coefficient and heat transfer rate decrease as the values of conduction parameter increase.

5. TABLES AND FIGURES

Table 1: Numerical values of $-\theta'(x,0)$ for different values of x while Pr =1.0, M = 0.0, ε = 0.0 and p = 0.0.

$Nu\,Gr^{-1/4} = -\theta'(x,0)$			
x	Merkin [1]	Nazar et al. [14]	Present
0.0	0.4214	0.4214	0.4216
$\pi/6$	0.4161	0.4161	0.4163
$\pi/3$	0.4007	0.4005	0.4006
$\pi/2$	0.3745	0.3741	0.3741
$2\pi/3$	0.3364	0.3355	0.3355
$5\pi/6$	0.2825	0.2811	0.2811
π	0.1945	0.1916	0.1912

Table 2: Numerical values of $x\,f''(x,0)$ for different values of x while Pr = 1.0, M = 0.0, ε = 0.0 and p = 0.0.

$C_f\,Gr^{1/4} = x\,f''(x,0)$			
x	Merkin [1]	Nazar et al. [14]	Present
0.0	0.0000	0.0000	0.0000
$\pi/6$	0.4151	0.4148	0.4139
$\pi/3$	0.7558	0.7542	0.7528
$\pi/2$	0.9579	0.9545	0.9526
$2\pi/3$	0.9756	0.9698	0.9678
$5\pi/6$	0.7822	0.7740	0.7718
π	0.3391	0.3265	0.3239

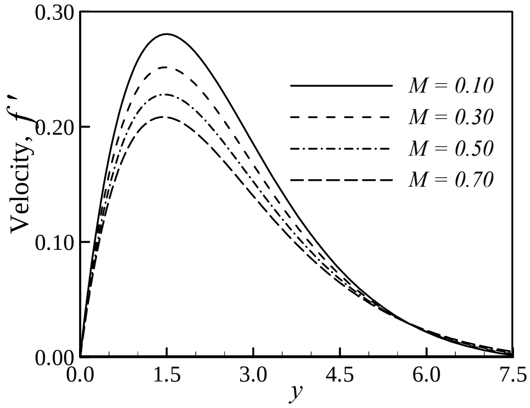

Fig.2. Variation of velocity profiles against y for varying of M with Pr = 1.0, ε = 0.01 and p =1.0.

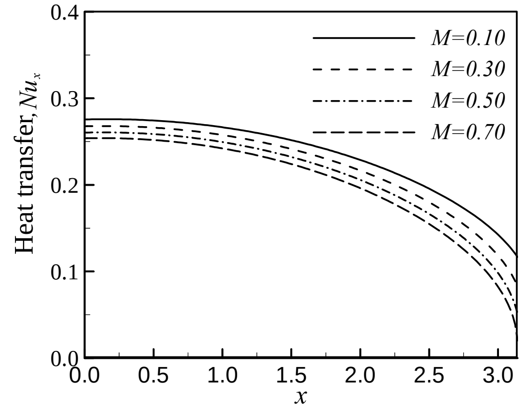

Fig.5. Variation of rate of heat transfer against x for varying of M with Pr = 1.0, ε = 0.01 and p =1.0.

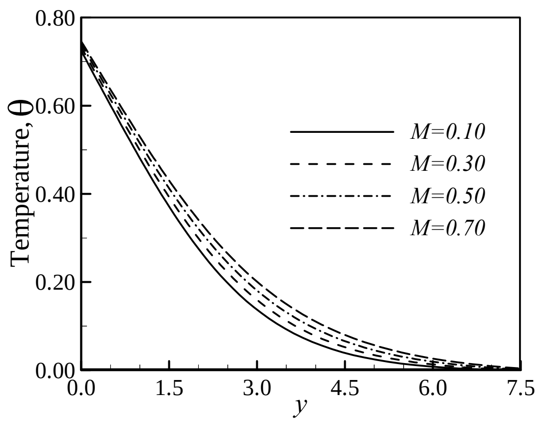

Fig.3. Variation of temperature distributions against y for varying of M with Pr = 1.0, ε = 0.01 and p =1.0.

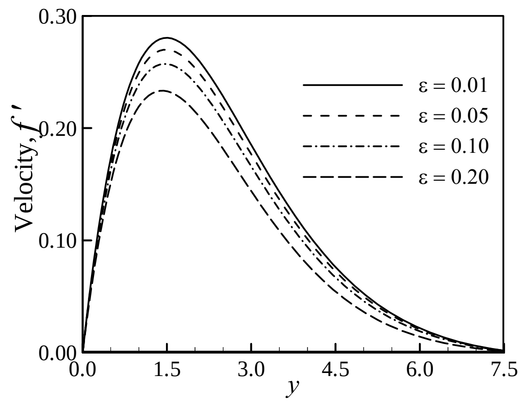

Fig.6. Variation of velocity profiles against y for varying of $\boldsymbol{\varepsilon}$ with Pr = 1.0, M = 0.1 and p = 1.0.

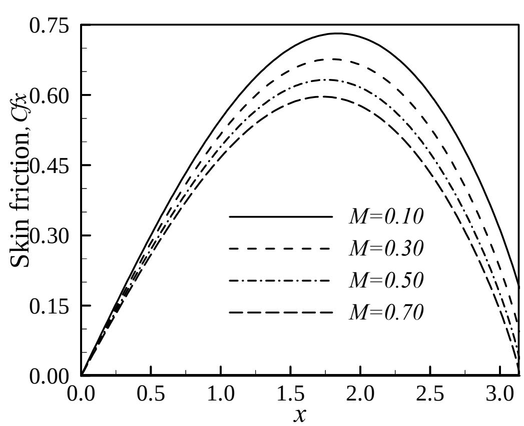

Fig.4. Variation of skin friction coefficients against x for varying of M with Pr = 1.0, ε = 0.01 and p=1.0.

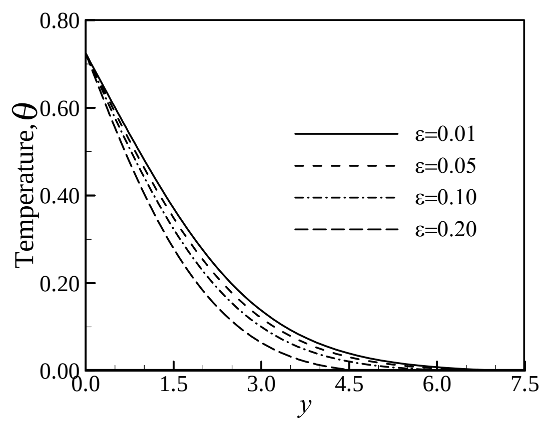

Fig.7. Variation of temperature distributions against y for varying of $\boldsymbol{\varepsilon}$ with Pr = 1.0, M = 0.1 and p = 1.0.

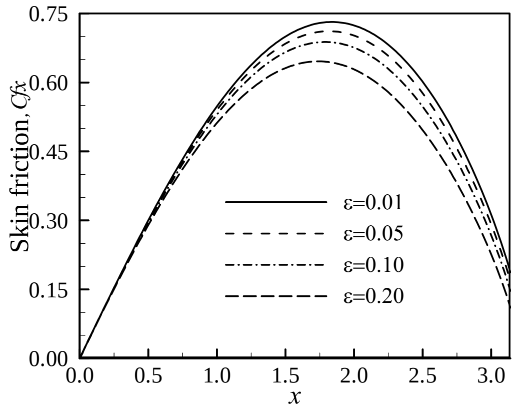

Fig.8. Variation of skin friction coefficients against x for varying of $\boldsymbol{\varepsilon}$ with Pr = 1.0, M = 0.1 and p =1.0.

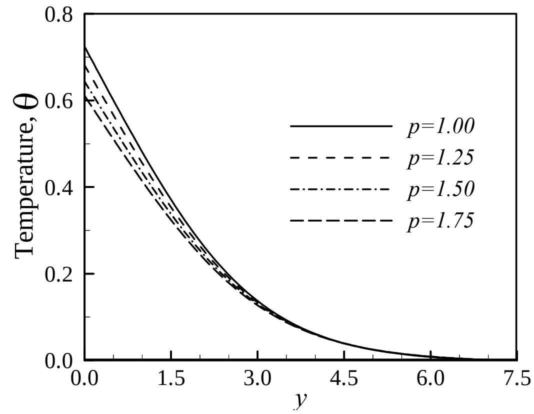

Fig.11. Variation of temperature distributions against y for varying of p with Pr = 1.0, M = 0.1 and ε = 0.01.

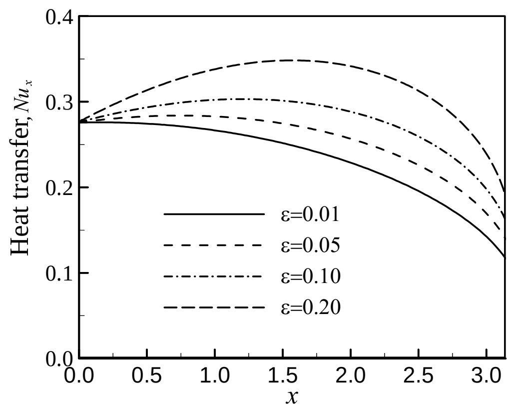

Fig.9. Variation of rate of heat transfer against x for varying of $\boldsymbol{\varepsilon}$ with Pr = 1.0, M = 0.1 and p =1.0.

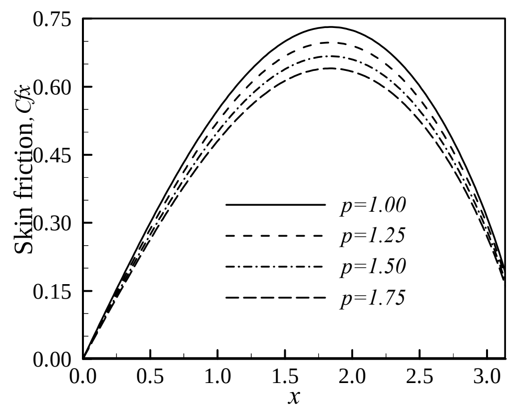

Fig.12. Variation of skin friction coefficients against x for varying of p with Pr = 1.0, M = 0.1 and ε = 0.01.

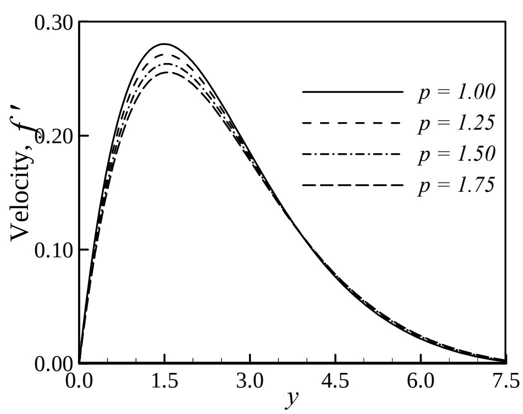

Fig.10. Variation of velocity profiles against y for varying of p with Pr = 1.0, M = 0.1 and ε = 0.01.

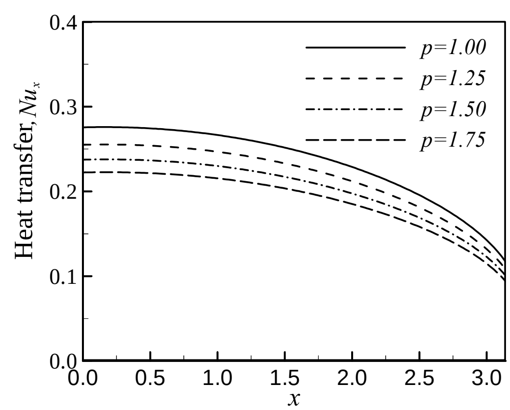

Fig.13. Variation of rate of heat transfer against x for varying of Pr with ε = 0.01, M = 0.5 and p = 1.0.

6. CONCLUSION

MHD-conjugate free convection flow from horizontal circular cylinder considering stress work effect is studied. The effects of the magnetic parameter, Stress work parameter and Conjugate conduction parameter are analysed on the fluid flow with Prandtl number Pr = 1.0. The velocity of the fluid within the boundary layer decreases with increasing magnetic parameter, stress work parameter and conjugate conduction parameter. The temperature distribution increases for increasing magnetic parameter while it decreases with increasing stress work parameter and conjugate parameter. The skin friction coefficient along the surface decreases for all three parameters however the rate of heat transfer increases for increasing stress work parameter while it decreases for increasing magnetic parameter and conjugate conduction parameter.

REFERENCES

[1] Merkin, J. H., "Free convection boundary layer on an isothermal horizontal cylinder", ASME, paper no.76-HT-16, (1976).

[2] Merkin, J. H. and Pop, I., "A note on the free convection boundary layer on a horizontal circular cylinder with constant heat flux", Wärme und Stoffübertragung, Vol. 22, pp.79-81 (1988).

[3] Kuehn, T. H. and Goldstein, R. J., "Numerical solution to the Navier-Stokes equations for laminar natural convection about a horizontal isothermal circular cylinder", Int. J. heat Mass Transfer, Vol.23, pp.971-979 (1980).

[4] Wang, P., Kahawita, R. and Nguyen, T. H., "Numerical computation of natural convection flow about a horizontal cylinder Using Splines", Numerical Heat Transfer, Part-A, Vol.17, pp.191-215(1990).

[5] Gdalevich, L. B. and Fertman, V. E., "Conjugate problems of natural convection", Inzh-Fiz. Zh. 33, pp.539–547(1977).

[6] Miyamoto, M., Sumikawa, J., Akiyoshi, T. and Nakamura,T., "Effects of axial heat conduction in a vertical flat plate on free convection heat transfer", International Journal of Heat and Mass Transfer 23, pp.1545–1553 (1980).

[7] Pozzi, A. and Lupo, M., "The coupling of conduction with laminar natural convection along a flat plate", International Journal of Heat and Mass Transfer 31, pp.1807–1814 (1988).

[8] Kimura, S. and Pop, I., "Conjugate natural convection from a horizontal circular cylinder", Numerical heat transfer, Part A, Vol.25, pp.347-361 (1994).

[9] Wilks, G., "Magnetohydrodynamics free convection about a semi-infinite vertical plate in a strong cross field", ZAMP, Vol.27, pp.621-631 (1976).

[10] Aldoss, T. K., Ali, Y. D. and Al-Nimr, M. A., "MHD mixed convection from a horizontal circular cylinder", Numerical Heat Transfer, Part A, Vol.30, pp.379-396 (1996).

[11] El-Amin, M. F., "Combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal cylinder embedded in a fluid saturated porous medium", Journal of Magnetism and Magnetic materials, Vol.263, pp.337-343 (2003).

[12] Gebhart, B., "Effects of viscous dissipation in natural convection," Journal of Fluid Mechanics, Vol. 14, pp.225–232, (1962).

[13] Pantokratoras, A., "Effect of viscous dissipation and pressure stress work in natural convection along a vertical isothermal plate. New results," International Journal of Heat and Mass Transfer , Vol. 46 pp. 4979–4983 (2003).

[14] Nazar, R., Amin, N. and Pop, I., "Free convection boundary layer on an isothernal horizontal circular cylinder in a micropolar fluid" Heat transfer, Proceeding of the 12th int. conference (2002).

[15] Keller, H. B., "Numerical methods in the boundary layer theory", Annual Reviews of Fluid Mechanics 10, pp.417–433 (1978).

[16] Cebeci, T. and Bradshaw, P., "Physical and computational aspects of convective heat transfer", Springer, New York (1984).

NOMENCLATURE

Symbol	Meaning	Unit
a	Radius of the outer cylinder	(cm)
b	Thickness of the cylinder	(cm)
B_0	Applied magnetic field	(N)
C_{fx}	Skin friction coefficient	…
c_p	Specific heat	(J/Kg.K)
f	Dimensionless stream function	…
g	Acceleration due to gravity	(cm/s^2)
M	Magnetic parameter	…
Nu_x	Local Nusselt number	…
p	Conjugate conduction parameter	…
Pr	Prandtl number	…
T_b	Temperature of the inner cylinder	(K)
T_f	Temp. at the boundary layer region	(K)
T_s	Temp. of the solid of the cylinder	(K)
T_∞	Temperature of the ambient fluid	(K)
$\bar{u},\ \bar{v}$	Velocity components	(cm/s)
u, v	Dimensionless velocity components	…
$\bar{x},\ \bar{y}$	Cartesian coordinates	(cm)
$x,\ y$	Dimensionless Cartesian coordinates	…
Greek symbols	Meaning	Unit
β	Co-efficient of thermal expansion	(K^{-1})
ε	Stress work parameter	…
ψ	Dimensionless stream function	…
ρ	Density of the fluid inside	(Kg/m^3)
ν	Kinematic viscosity	(m^2/s)
μ	Viscosity of the fluid	(N.s/m^2)
θ	Dimensionless temperature	…
σ	Electrical conductivity	J/msK
K_f	Thermal conductivity of the fluid	(kW/mK)
K_s	Thermal conductivity of the solid	(kW/mK)