NUMERICAL INVESTIGATION ON NATURAL CONVECTION FROM AN OPEN RECTANGULAR CAVITY CONTAINING A HEATED CIRCULAR CYLINDER

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ABSTRACT

The problem of natural convection heat transfer in a square open cavity containing a heated and conducting circular cylinder at the centre is analyzed in this paper. As boundary conditions of the cavity, the left vertical wall is kept at a constant heat flux, bottom and top wall are kept at different high and cold temperature respectively. The remaining side is open. Two dimensional laminar steady state natural convection is considered. This configuration is related in the design of electronic devices, solar energy receivers, uncovered flat plate solar collectors, geothermal reservoirs etc. The fluid is concerned with different Prandtl numbers; Grashof numbers and the properties of the fluid are assumed to be constant. The development of Mathematical model is governed by the coupled equations of continuity, momentum and energy and is solved by employing Galerkin weighted finite element method. Flow field and heat transfer were predicted for fluid with Pr = 0.72 , 1.0 ,7.0; Gr = 10^3,10^4, 10^5 ,10^6; inclination angles of the cavity are φ = 0^o, 15^o,30^o, 45^o and cylinder diameter to cavity length ratio dr = 0.2.The average Nusselt number increases as the increases of inclination angle of the cavity for lower Pr and lower temperature at bottom wall. The average Nu increases mainly for higher inclinations and for higher Gr. Various vortexes and recirculations are formed into the flow field for higher Pr and higher temperature at the bottom wall.

Key words*: Heat transfer, finite element method, natural convection, square open cavity, heated circular* cylinder.

NOMENCLATURE

C_{fx} Local skin friction coefficient
C_p Specific heat at constant pressure
f Dimensionless stream function
g Gravitational acceleration (ms^{-2})
Gr Grashof number
K Thermal conductivity of the fluid ($Wm^{-1}K^{-1}$)
H Height
L Length of the cavity (m)
D Diameter of the cylinder
Nu Nusselt number
P pressure (Nm^{-2})
P non-dimensional pressure
Pr Prandtl number, υ/α
Q Heat flux (Wm^{-2})
dr cylinder diameter to cavity length ratio
T Temperature of the fluid in the cavity (K)
T_s Temperature at the surface (K)
T_∞ Temperature of the ambient fluid (K)
T_h High temperature at the bottom wall (K)
T_c Cold temperature at the upper wall (K)
Nu_{av} Average Nusselt number
u,v Velocity component (ms^{-1})
U_∞ Free stream velocity(ms^{-1})
U_0 Reference velocity
U,V non-dimensional velocity components
x,y Cartesian co-ordinates (m)
X,Y non-dimensional Cartesian co-ordinates
T_{cl} temperature of the cylinder

1. INTRODUCTION

The study of natural convection in open cavities has been the subject of many experimental and numerical investigations during the past two decades. The previous researchers have investigated the effect on flow and heat transfer for different Rayleigh numbers, aspect ratios and tilt angles. As a result a number of studies have been done for natural convection heat transfer over a cylinder placed inside a closed enclosure.

Showole and Tarasuk (1993) studied experimentally and numerically, the steady state natural convection in a two dimensional isothermal open cavity. They found the experimental results for air, varying the Rayleigh number from 10^4 to 5.5×10^5, cavity aspect ratios of 0.25, 0.5 and 1.0 and inclination angles of 0^o, 30°, 45^o and 60° (for 0°, the wall opposite the aperture was horizontal and the angles were taken clockwise). They calculated the numerical results for Raleigh numbers between 10^4 and 5.5×10^5, inclination angles of 0° and 45° and an aspect ratio equal to one. In the result it is seen that for all Rayleigh numbers, the first inclination of the cavity caused a significant increase in the average heat transfer rate, but a further increase in the inclination angle caused very little increase in the heat transfer rate. In another result it is observed that, for 0°, two symmetric counter rotating eddies were formed, while at inclination angles greater than 0°, the symmetric flow and temperature patterns disappear.

Chan and Tien (1985a) investigated a square open cavity numerically, which had an isothermal vertical heated side facing the opening and two adjoining adiabatic horizontal sides. To obtain the satisfactory solutions in the open cavity the boundary conditions for field were approximated.

Chan and Tien (1985b) investigated shallow open cavities and made a comparison study using a square cavity in an enlarged computational domain. In the result they observed that for a square open cavity having an isothermal vertical side facing the opening and two adjoining adiabatic horizontal sides. Satisfactory heat transfer results could be obtained, especially at high Rayleigh numbers.

Mohammad (1995) investigated inclined open square cavities, by considering a restricted computational domain. The gradients of both velocity components were set to zero at the opening plane in that case which were different from those of Chan and Tien (1985a). In the result he found that heat transfer was not sensitive to inclination angle and the flow was unstable at high Rayleigh numbers and small inclination angles.

Rahman et al. (2009) investigated on Mixed Convection in a rectangular cavity with inlet, outlet and a heat conducting Horizontal Circular Cylinder. They found that both the heat transfer rate from the heated wall and the dimensionless temperature in the cavity strongly depends on the governing parameters and configurations of the system such as size, location, and thermal conductivity of the cylinder and location of the inflow, outflow opening.

2. MODEL SPECIFICATION

2.1 Physical Model

The physical model considered here along with necessary geometric parameters. In this study heat transfer and the fluid flow in a two-dimensional open rectangular cavity of length L was considered, which is shown in the following schematic diagram.

2.2 Mathematical Formulation

The flow inside the cavity is assumed to be two-dimensional, steady, laminar, and incompressible and the fluid properties are to be constant. The radiation effects are taken negligible and the Boussinesq approximation is used. The dimensionless governing equations describing the flow are as follows:

Continuity equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

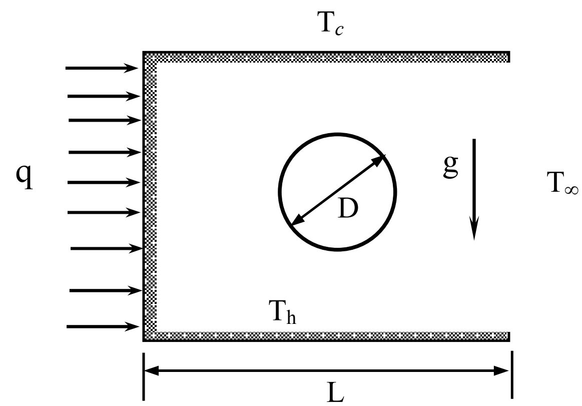
Momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\theta\sin\phi$$

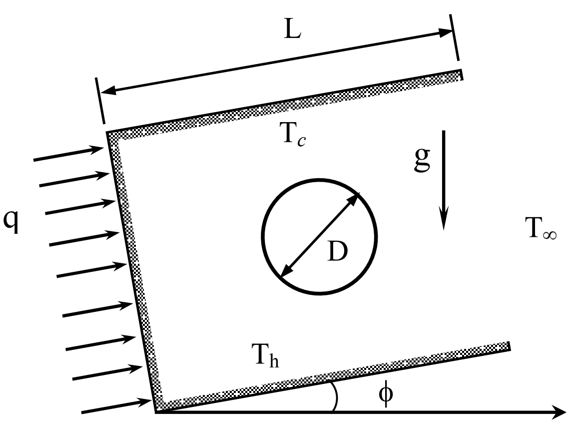
$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\theta\cos\phi$$

Energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)$$



(a) Horizontal cavity



(b) Inclined cavity

Figure 1. Schematic diagram of the physical problem

2.2.1 Boundary Conditions

At the bottom wall: $U=V=0;\ \theta=1$

At the top wall: $U=V=0\ ;\ \theta=0$

At the left wall: U =V = 0 ; heat flux q = 500

At the right side & open side:

Convective Boundary Condition (CBC), P = 0

Non-dimensional scales:

$$X = \frac{x}{L},\quad Y = \frac{y}{L},\quad U = \frac{u}{U_0},\quad V = \frac{v}{U_0},$$

$$P = \frac{P - P_\infty}{\rho\ U_0^2}, \theta = \frac{T - T_\infty}{\Delta t}, \Pr = \frac{\nu}{\alpha}, Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

$$dr = \frac{D}{L},\qquad \Delta t = \left(T_s - T_\infty\right)\ , \Delta t = \frac{qL}{K}$$

2.3 Used Heat Transfer Parameter

Since the flow field in natural convection is governed by the dimensionless Grashof Number Gr. Where the Grashof number represents the ratio of the buoyancy force to the viscous force acting on the fluid and the reference velocity U_o is related to the buoyancy force term and is defined as

$$U_{\ 0} = \sqrt{g\beta L\left(T_s - T_\infty\right)}$$

The Nusselt number Nu is also an important non-dimensional parameter to be computed for heat transfer analysis in natural convection flow. The Nusselt number for natural convection is a function of the Grashof number only. The local Nusselt number Nu can be obtained from the temperature field by applying the function

$$Nu = -\frac{1}{\theta(0,Y)}$$

The overall or average Nusselt number was calculated by integrating the temperature gradient over the heated wall as follows:

$$Nu_{av} = -\int_0^1 \frac{1}{\theta(0,Y)} dy$$

Since the dimensionless Prandtl Number Pr is the ratio of kinematic viscosity to thermal diffusivity. So Pr is a heat transfer parameter in the flow field of natural convection.

2.4 Grid Independence Test

To obtain grid independent solution, a grid refinement study is performed for a rectangular open cavity with Gr=10^6 and dr=0.2.The test shows the convergence of the average Nusselt number Nu at the heated surface with grid refinement. It is observed that grid independence is achieved with 13686 elements where there is insignificant change in Nu with further increase of mesh elements. The grid refinement tests are done by taking six different non–uniform grids with the following number of nodes and elements: 27342 nodes, 4818 elements; 49335 nodes, 7663 elements; 72782 nodes, 10365 elements; 73542 nodes, 11413 elements; 96030 nodes, 12356 elements; 892450 nodes, 13686 elements. In this case 982450 nodes, 13686 elements can be chosen through the processing to optimize the relation between the accuracy required and the computational time.

2.5 Code Validation

Due to lack of availability of experimental and numerical results published earlier on this regard, validation of this prediction could not be done. The present numerical result is verified against documented numerical study, namely; Finite element simulation of natural convection from an open rectangular cavity containing adiabatic circular cylinder– the M. Phil. Thesis in Mathematics by Koabra (2008) BUET, Dhaka.

Table1. Results for the different Grashof numbers with Pr = 0.72, dr = 0.2

Gr	**Nu_{av}**	
	Present work	**Koabra(2008)**
10^3	3.3282	3.2473
10^4	3.4014	3.3020
10^5	4.4181	4.1825
10^6	5.8227	5.5521

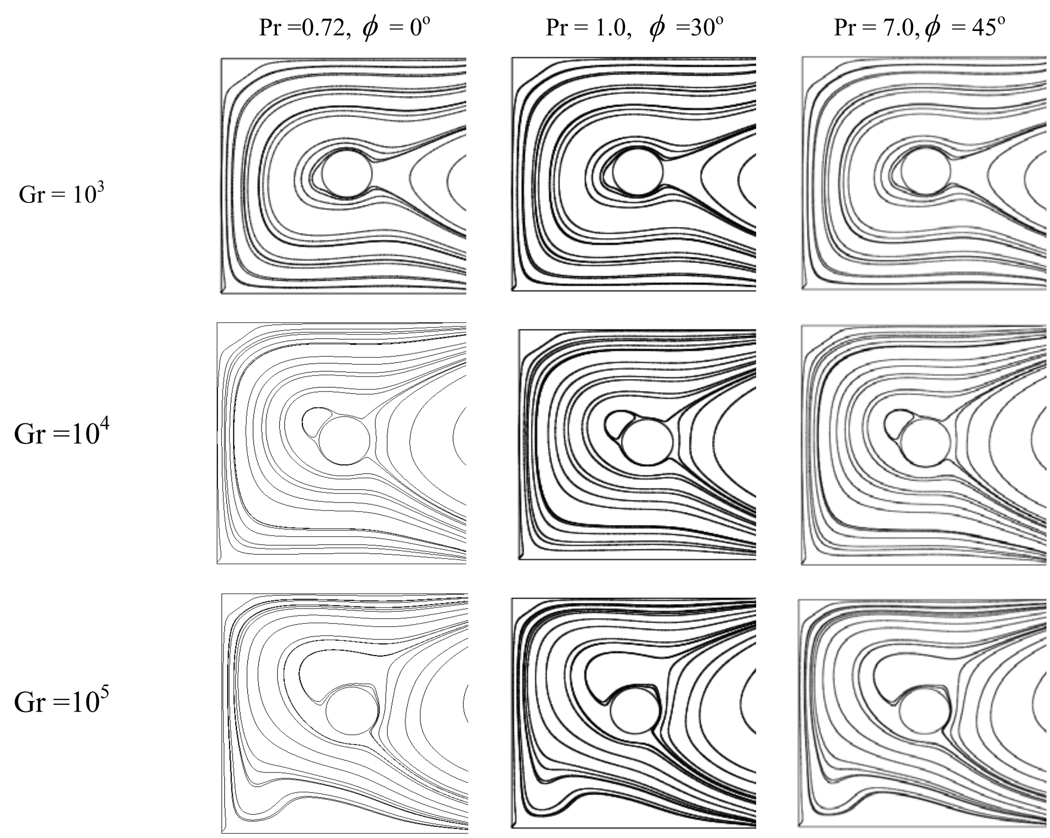
3. RESULTS AND DISCUSSIONS

The results Nu_{av} are obtained for Grashof number from 10^3 to 10^6 at Pr = 0.72, 1.0, 7.0 while inclination angles 0^o to 45^o and different high temperatures 320k, 350k , 375k at bottom wall with constant physical properties. Here the parametric analysis for a wide range of governing parameters shows consistence performance of the present numerical approach to obtain as streamlines and temperature profiles. The obtained results show that the heat transfer coefficient Nu is strongly affected by Grashof number. An empirical correlation can be developed using Nusselt number and Grashof number. In the effect of Grashof number, the isotherm figures show that as the Grashof number and the inclination angle increases the buoyancy force increases and the thermal boundary layers become thinner in Figures 2 and3. The Figures 2 and 3 of streamlines and isotherms for Pr = 0.72, 1.0, 7.0, high temperature T_h =320k at the bottom wall, cold temperature T_c = 276 k at the upper wall and T_{cl} =300k temperature of the cylinder show that the fluid moves from the bottom wall of the cavity, circulates in a clockwise direction around the cylinder and moves toward the upper part of the cavity and fresh cool fluid enter the cavity through the bottom wall. The streamlines patterns are very similar for different inclinations while Gr = 10^3. The fluid moves faster for G=10^4 to 10^6 and the upper thermal boundary layer becomes thinner. One vortex is formed at the centre of the opening side for each of Gr =10^3 to

10^6, because of the effect of the heated circular cylinder. One vortex is formed near the cylinder for each of Gr = 10^4, 10^5 and two vortexes are formed for Gr =10^6 near the bottom wall. Streamlines show that as the inclination angle of the heated wall increases, the velocity gradient remained almost same as before for Pr = 0.72, 1 and 7.0. The variations of the average Nusselt number and the average temperature are also presented. The results are obtained for a Grashof numbers ranging from 10^3 to 10^6 and for inclination angles range from 0^o to 45^o of the cavity. In figures of streamlines are almost laminar for Gr = 10^3 while Pr = 7.0. Due to the effect of the heated cylinder, a vortex is formed at the open side of the cavity for each of Gr = 10^3 to 10^6. Due to increase in T_h at the bottom wall, the fluid rises faster from the bottom wall and one vortex is created for each of Gr = 10^4, 10^5 near the cylinder, one small vortex is formed near the bottom wall while Gr = 10^5. Two vortexes are formed near the bottom wall for Gr =10^6 .The streamlines and isotherms for dr = 0.2, Pr = 1.0, T_h = 375 K, T_c = =278 K and for inclination angles ϕ = 0^o, 30^o, 45^o are shown in Figure 4 and Figure 5 respectively for the variation of Grashof number. One vortex is created for each of Gr = 10^3, 10^4, 10^5, 10^6 on the open side. This unsteady streamlines and isotherms are occurred due to higher Prandtl number Pr = 7.0. Since the relation between Nu and Pr is non-linear and one vortex is formed near the bottom wall for Gr = 10^6. The isotherms patterns of these Figures are similar.

For Grashof number 10^3, the streamlines are almost laminar but one vortex is created for each of Gr = 10^4, 10^5; but two recirculation cells are formed near the cylinder and bottom wall for Gr = 10^6. One small vortex is formed at the bottom wall. One vortex is created near the cylinder for each of Gr = 10^3, 10^4.

Two recirculation are formed for Gr = 10^5,10^6 one of these is near the cylinder and another is near the bottom wall. One vortex is formed on the open side for each of Gr =10^3, 10^4, 10^5 and 10^6. This is due to increase in temperature at the bottom wall and presence of the highly heated circular cylinder and cylinder redirect the fluid. One vortex is formed on the open side for each of Gr =10^3, 10^4, 10^5 and 10^6. This is due to increase in temperature at the bottom wall and presence of the highly heated circular cylinder and cylinder redirect the fluid. Some streamlines and isotherms patterns of this problem are shown in the following Figures from Figure 2 to Figure 5.



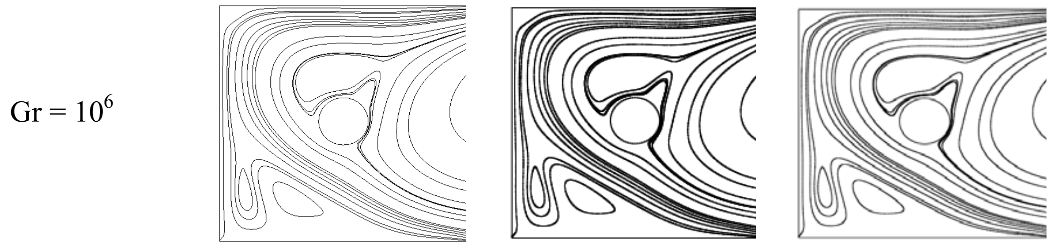


Figure 2: Streamlines for dr = 0.2, Gr = 10^3, 10^4 , 10^5, 10^6 and T_h =320k

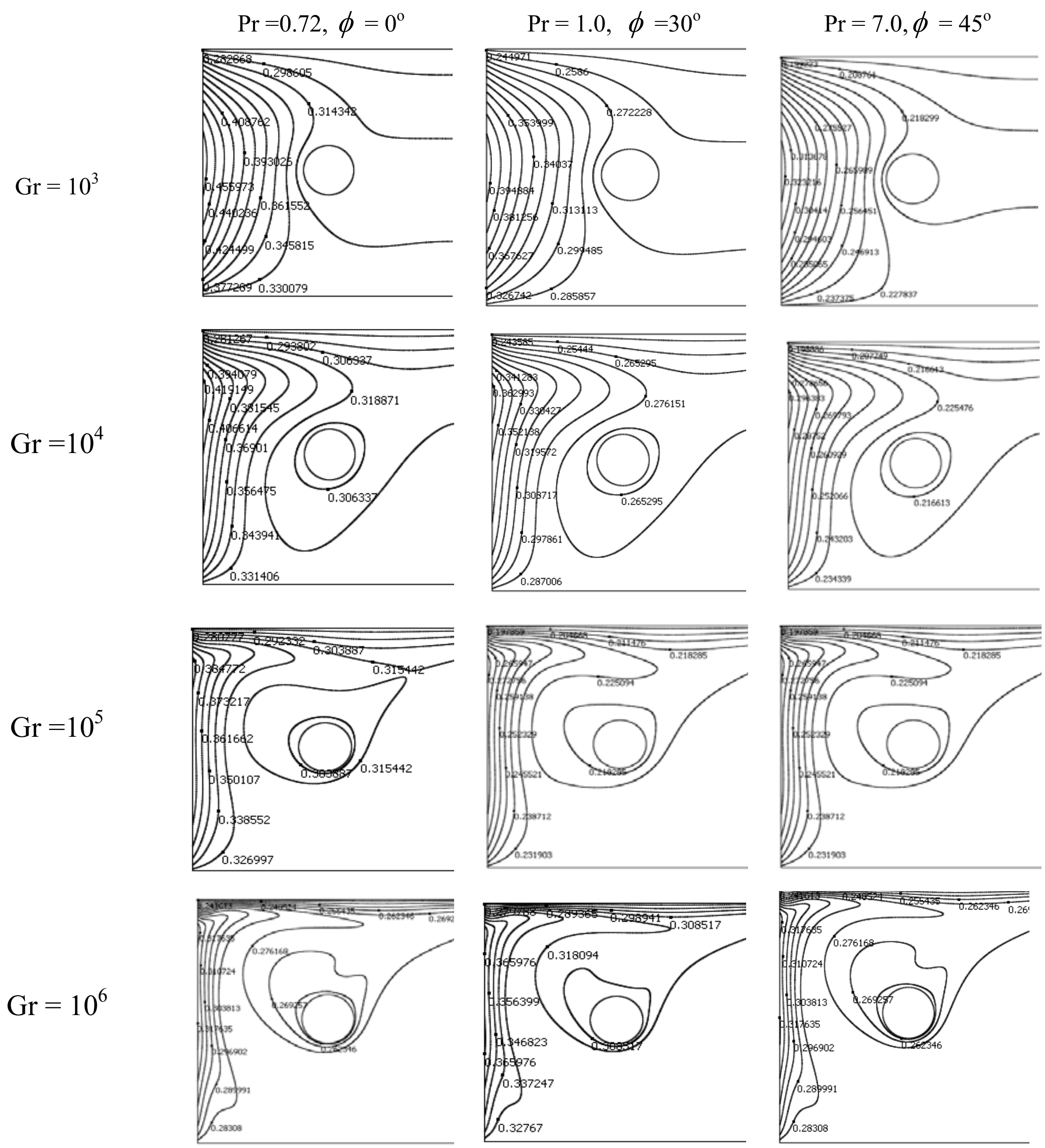


Figure 3: Figures of Isotherms for dr = 0.2, Gr = 10^3, 10^4, 10^5, 10^6 and T_h = 320

Table 2**.** Average Nusselt number Nu_{av} for different inclination angles ϕ and Grashof numbers with Pr = 0.72 and dr = 0.2.

ϕ	Nu_{av}			
	Gr=10^3	Gr=10^4	Gr=10^5	Gr=10^6
0^o	3.2725	3.29	4.07	5.4947
15^o	3.2756	3.37	4.2837	5.8547
30^o	3.3525	3.5443	4.5646	5.9657
45^o	3.4125	3.4956	4.7543	5.9757

Table 3. Average Nusselt numbers (Nu_{av}) for different Prandtl numbers and Gr taking Pr = 0.72, 1.0 and 7.0, angle ϕ =0^o and dr =0.2

Pr	Nu_{av}			
	Gr =10^3	Gr =10^4	Gr =10^5	Gr =10^6
0.72	3.2825	3.2975	4.2575	5.6747
1.00	3.2734	3.3595	4.3465	5.8857
7.00	3.3255	4.4231	4.8753	8.8447

5. CONCLUSION

The flow field and heat transfer are studied in a square open cavity containing a heated circular cylinder of diameter ratio dr = 0.2. The effect of Pr , Gr and high temperature at the bottom wall are studied for various inclination angle for the cavity. In this study we observed that the heat transfer rate depends on Prandtl number and Grashof number. Heat transfer rate increases as Prandtl number increases. Thermal boundary layer thickness becomes thinner for higher Grashof number. The heat transfer rate decreases for Gr =10^3 and increases gradually for increasing of Grashof number. The heat transfers rate Nu increases as inclination angle of the cavity and Grashof number increase. Various vortexes and recirculation are formed into the flow field and a boundary vortex at the centre of the cavity is seen in the streamlines.

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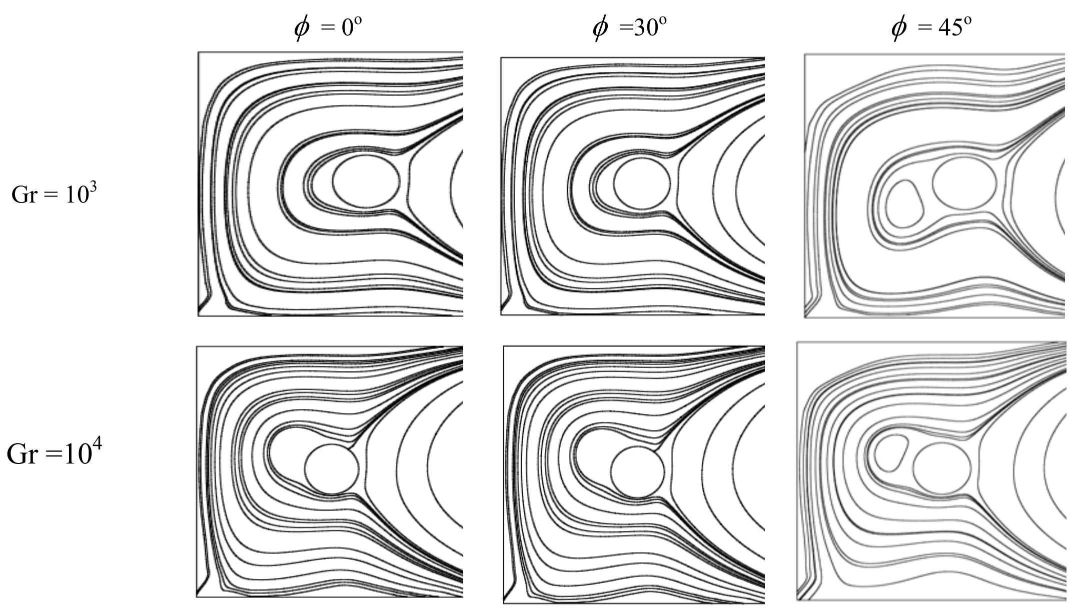


Gr $=10^5$

Gr $= 10^6$

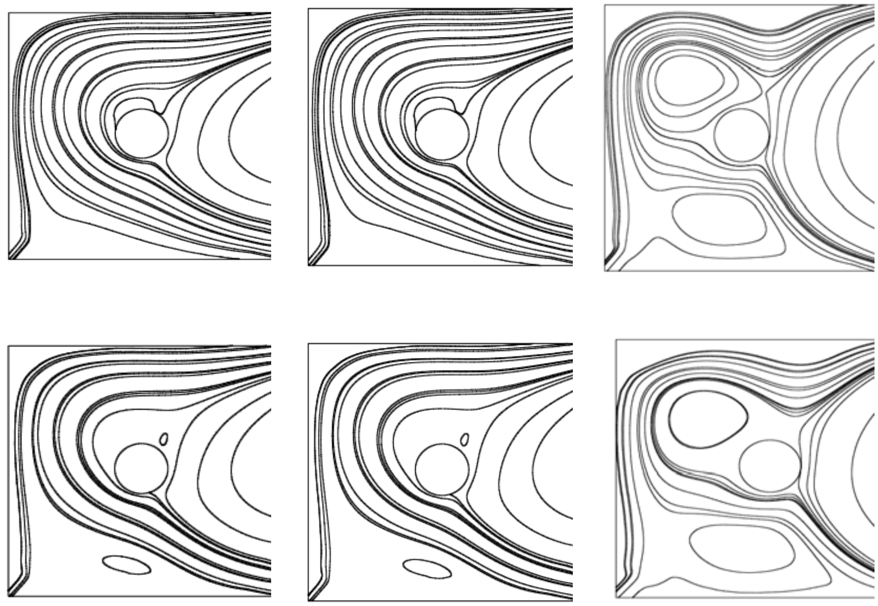


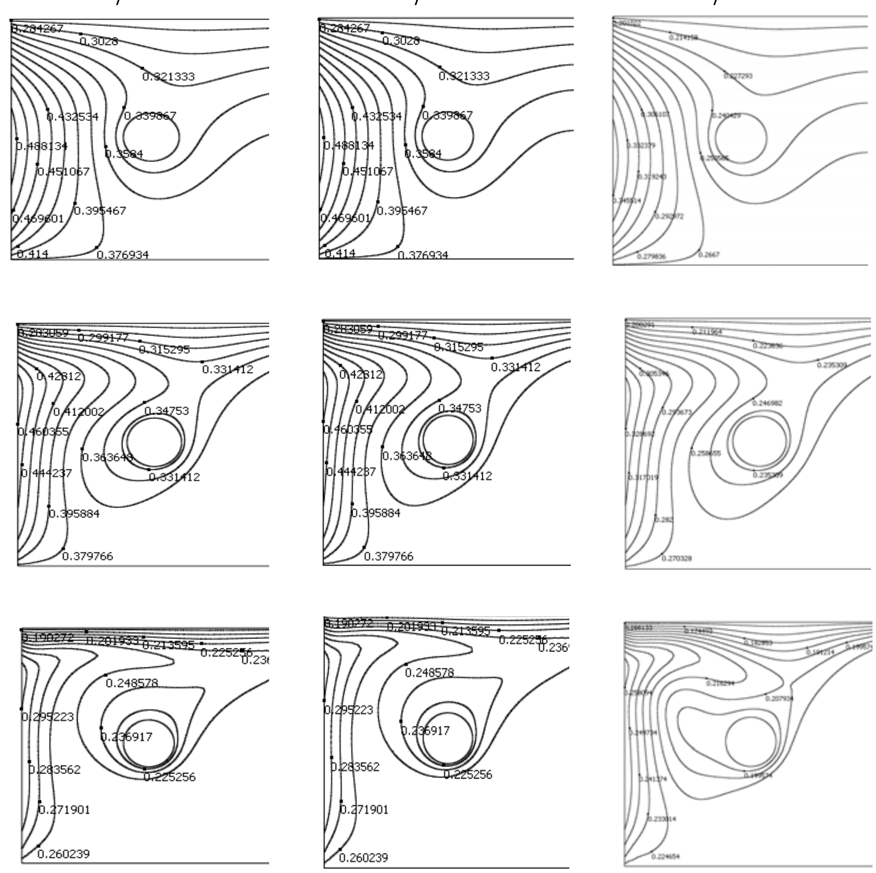
Figure 4. Streamlines patterns for dr = 0,2 , Pr = 1.0.and $T_h = 375$ k

ϕ = 0^o ϕ =30^o ϕ = 45^o

Gr = 10^3

Gr $=10^4$

Gr $=10^5$



Gr = 10^6

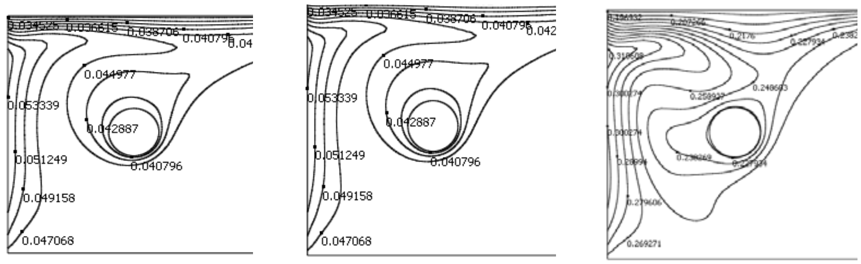


Figure 5: Isotherms patterns for dr = 0,2 , , Pr = 1.0. and T_h = 375 k

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