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MOTION RESPONSES AND INCIDENT WAVE FORCES ON A MOORED SEMI SUBMERSIBLE IN REGULAR WAVES

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ABSTRACT

Nowadays, floating structures play an important role for exploring the oil and gas from the sea. With ever-growing needs for oil and gas resources, the ocean engineering has been paid much attention to the world engineering community not only in coastal regions but also in deepwater. Unlike seagoing ships, moored floating offshore structures such as semi submersibles and tension leg platforms (TLP) are usually positioned at a given location at sea and their motion is externally constrained by the moorings. Motion response of a floating structure should be kept adequately low to guarantee the safety of risers and umbilical pipes as most important components in the equipment of oil production. The objectives of this study is to investigate the hydrodynamic forces and motions of both free floating and moored semi submersible under incident, scattered and radiated waves numerically. In numerical formulation, hydrodynamic problems are solved by using three-dimensional source distribution method, within the scope of linear wave theory and using frequency domain. Since experimental test have not been done yet, so results obtained from computations were validated with the results obtained using commercial software MOSES and WAMIT and other published papers.

Key words: semi submersible, floating structures, 3D source density distribution technique

1. INTRODUCTION

Floating structures such as ship, semi-submersible, FPSO, TLP, breakwater and other free floating or moored structures, are subjected to wave, wind and current at sea. They have six-coupled degrees of freedom of motions. Namely, linear motions are surge, sway and heave, and angular motions are roll, pitch and yaw. Oscillation of floating structure affects the loading and offloading operation systems.

There are different theories for studying motion of floating structure such as strip theory and potential theory. In this paper 3D source density distribution technique is used to get the potential over the floating structure. Having flow velocity potentials on and off the panels, hydrodynamic coefficients of floating structure can be determined. Using Bernoulli's equation leads to calculation of pressure distribution and forces over the floating structure. A mathematical model is mathematical structure that can be used to describe and study a real situation. A second-order linear differential equation for coupled six degree of freedom can describe the hydrodynamics of floating structures; consist of added mass, damping coefficient, stiffness coefficient, forces and motions in six directions.

J. L. Hess and A. Smith [1] studied on the calculation of non-lifting potential flow about arbitrary 3D bodies. They utilized a source density distribution on the surface of the body and solved for distribution necessary to take the normal component of fluid velocity zero on the boundary. Plane quadrilateral source elements were used to approximate the body surface, and the integral equation for the source density is replaced by a set of linear algebraic equations for the values of the source density on the quadrilateral elements. By solving this set of equations, the flow velocities both on and off the surface were calculated.

Oortmerssen [2] dealt with the hydrodynamic forces between two structures floating in waves by using a three-dimensional linear diffraction theory and the results agree well with experiments..

Wu et al. [4] studied the motion of a moored semi in regular waves and wave induced internal forces numerically and experimentally. In the mathematical formulation, they modeled the moored semi as an externally constrained floating body in waves, and derived the linearized equation of motion.

Yilmaz and Incecik [3] analyzed the extreme motion response of moored semi-submersible. They developed and employed two different time domain techniques since there are strong nonlinearities in the system due to mooring line stiffness and damping and viscous drag forces. First one is for simulation of wave frequency motions in which the first-order wave forces are the only excitation forces. First-order wave forces acting on semi-submersibles are evaluated according Morison equation, current effect is taken into account by altering the drag term in Morison equation. Second one is to simulate the slowly varying and steady motions under the excitation of slowly varying wave, current and dynamic wind forces. Slowly varying wave forces are calculated using the mean drift forces in regular waves and applying an exponential distribution of the wave force record in irregular waves.

Soylemez [5] developed a prediction technique to simulate the motion response of damaged platform under wave, wind and current forms. The equation of motion was obtained using Newton's second law and the numerical solution technique of non-linear equations of motion is explained for intact and damaged cases. The analysis technique employs large displacement non-linear equations of motion. Solutions were obtained in the time domain to predict the motion characteristics.

Clauss et al [8] analyzed numerically and experimentally the sea-keeping behavior of a semi submersible in rough waves in the North Sea. They used panel method TiMIT (Time-domain investigations, developed at the Massachusetts Institute of Technology) for wave/structure interactions in time domain. The theory behind TiMIT is strictly linear and thus applicable for moderate sea condition only.

Sujatha and Soni [9] used diffraction theory to formulate the boundary value problem and analyzed forces and motion responses of moored offshore floating structures like semi submersible. They solved the interaction problem involves the motion of structure by using constant triangular elements on indirect BEM.

In this study the impact of regular waves is investigated for the semi submersible of the type of GVA 4000 which is characterized by favorable sea-keeping behavior. This offshore structure is designed for world wide operation, especially for the harsh conditions in the North Sea.

2. MATHEMTICAL MODEL

2.1 Coordinate System

The individual semi submersible is treated as a rigid body having six degrees of freedoms. It is subjected to hydrodynamic forces due to incident waves and radiated and diffracted waves due to other vehicle(s). Two right hand coordinate systems are defined in Figure 1. One is fixed to the space on water surface and the other one is fixed to the centre of gravity.

The fluid is assumed to be incompressible, inviscid and irrotational and the vessel is assumed to be freely floating in open water. Then there exists a velocity potential satisfying Laplace equation together with boundary conditions on the free surface, on the body, and at the bottom, and the radiation condition in the far field. The time dependence of the fluid motion to be considered here is restricted to simple harmonic motion and accordingly the flow filed can be characterized by the following velocity potential:

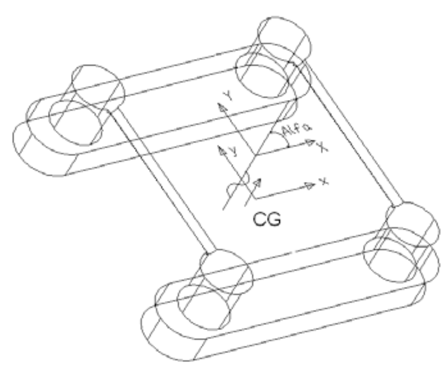

Figure 1: Definition of co-ordinate system.

$$\Phi = \mathrm{Re}[\phi(x, y, z)e^{-i\omega t}] \tag{1}$$

$$\phi = -i\omega\left[(\phi_0 + \phi_7).\zeta_a + \sum_{j=1}^{6}(X_j\phi_j)\right] \tag{2}$$

$$\phi_0 = \frac{-ig\zeta_a}{\omega}\frac{\cosh[k(z+h)]}{\cosh kh}e^{ik(x\cos\alpha + y\sin\alpha)} \tag{3}$$

Where,
φ_0=incident wave potential
φ_7= diffraction wave potential on body
φ_j= potential due to motion of the body in *j*-th mode
ω= circular frequency of incident wave
ζ_a=incident wave amplitude
α= wave heading angle from X -axis

The differential equation governing the fluid motion follows from the application of the continuity equation which yields the Laplace equation. The individual potentials are the solutions of the following Laplace equation:

$$\nabla^2\phi = 0 \qquad (4)$$

2.2 Boundary Condition

On the mean wetted surface area of body S, the above linear velocity potentials must satisfy the Laplace equation and also the following boundary conditions:

-linearized free surface condition:

$$\frac{\partial\phi}{\partial z} + \frac{\omega^2}{g}\phi = 0 \quad , \quad \text{at} \quad z=0, \qquad (5)$$

-boundary condition on the sea floor:

$$\frac{\partial\phi}{\partial z} = 0 \quad , \quad \text{on} \quad z = -h , \qquad (6)$$

Another boundary condition is the kinematics boundary condition on wetted surface of the floating bodies. Due to linearization, this boundary condition may be applied on the wetted surface (S) of the floating body in its equilibrium position:

$$\frac{\partial\phi_k}{\partial n} = -i\omega n_k \ , \ k = 1,2,...,7 \qquad (7)$$

$$\frac{\partial\phi_1}{\partial n} = -i\omega \ \cos(n,x)$$

$$\frac{\partial\phi_2}{\partial n} = -i\omega \ \cos(n,y)$$

$$\frac{\partial\phi_3}{\partial n} = -i\omega \ \cos(n,z)$$

$$\frac{\partial\phi_4}{\partial n} = -i\omega[(y-y_G)\cos(n,z)-(z-z_G)\cos(n,y)]$$

$$\frac{\partial\phi_5}{\partial n} = -i\omega[(z-z_G)\cos(n,x)-(x-x_G)\cos(n,z)]$$

$$\frac{\partial\phi_6}{\partial n} = -i\omega[(x-x_G)\cos(n,y)-(y-y_G)\cos(n,x)]$$

$$\frac{\partial\phi_7}{\partial n} = -\frac{\partial\phi_0}{\partial n}$$

$x_G, y_G, z_G =$ Co-ordinate of the centre of gravity of the body

$x, y, z =$ Investigating point on the wetted surface of the body

In order to ensure that the velocity potential has the correct amplitude behavior in the far field, the radiation and scattering potentials must satisfy the radiation (Summerfield) condition:

$$\frac{\partial\phi_k}{\partial r} - \frac{i2\pi}{L}\phi_k = 0 \ , \ \text{as}\ r \to \infty \quad k = 1,2,...,7 \qquad (8)$$

2.3 Velocity Potential

However, there is no analytical solution for φ_7 and φ_j, so the problem should be solved numerically. According to the 3-D source sink method, the potentials φ_7 and φ_j can be expressed in terms of well known Green functions that can be expressed by the following equation:

$$\phi_j(x,y,z) = \frac{1}{4\pi}\sum_{n=1}^{N}\iint_s \sigma_j(\xi,\eta,\zeta)G(x,y,z;\xi,\eta,\zeta)ds \qquad (9)$$

Where, (ξ,η,ζ) denotes a point on surface S and σ(ξ,η,ζ) denotes the unknown source distribution. The integral is to be carried out over complete immersed surface of the object. The Green function G (source potential) must in order of the representation in equation (9) to be valid, satisfy all the boundary conditions of the problem with the exception of the body boundary conditions and have a source like behavior. As a result, boundary conditions are reduced only to on wetted surfaces of the bodies. So, the wetted surfaces should be subdivided into panels to transform integral equations to a system of algebraic equations to determine unknown source density over each panel. The appropriate Green function used in this paper to the boundary value problem posed is given by Wehausen and Laitone [7]. After getting the source density, the velocity potentials on each panel can be obtained using the equation (9).

2.4 Forces and Moments

Once the velocity potential is obtained, the hydrodynamic pressure at any point on the body can be obtained from the linearized Bernoulli's equation and can be written as:

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + \frac{P}{\rho} + gz = 0 \qquad (10)$$

Now after putting the value of Φ in the equation (10), the following expression is obtained,

$$-\frac{P}{\rho} = -i\omega\phi + \frac{1}{2}(\nabla\phi)^2 + gz \qquad (11)$$

By neglecting the higher order terms, we can write:

$$P = -\rho gz + i\rho\omega\phi \qquad (12)$$

As first part of equation (12) is associated with the hydrostatic and steady forces, so neglecting this part, the first order wave exciting forces or moments and oscillatory forces and moments caused by the dynamic fluid pressure acting on the body can be obtained from the following integrals:

$$F_k .e^{-i\omega t} = -i\rho\omega e^{-i\omega t}\int_s\{\phi_0 + \phi_7\}.n_k ds \quad (13)$$

$$F_{kj} .e^{i\omega t} = -\rho e^{-i\omega t}\int_s\{\omega^2 \overline{X}_j \phi_j\}.n_k .ds \quad (14)$$

Where, F_k denotes the *k*-th component of wave exciting forces or moments, F_{kj} denotes the *k*-th component of force arising from the *j*-th component of motion of the body. Moreover, it is customary to decompose the hydrodynamic forces resulting from motion of the bodies into components in phase with the acceleration and velocity of the rigid body motions. These yield the added mass and damping coefficients respectively. These coefficients can be expressed from equation as:

$$a_{kj} = -\rho.\mathrm{Re}\left[\iint_S \phi_j .n_k ds\right] \quad (15)$$

$$b_{kj} = -\rho\omega.\mathrm{Im}\left[\iint_S \phi_j .n_k ds\right] \quad (16)$$

2.5 Equation of Motion

After solving the above exciting forces, added mass and damping coefficients, the motions of two ships can be solved by the following coupled equations of motions. To describe the motion of the multiple floating bodies, two co-ordinate systems, one fixed to the body and the other fixed to the space have been introduced. The two co-ordinate systems are shown in figure 1. The equation of motion is expressed by the time varying relation between these two co-ordinate systems. The equation of motion will be coupled dynamically because of hydrodynamic interaction and mechanical connections between them. So the equation can be considered by using the following matrix relationship:

$$\sum_{j=1}^{6}(M_{kj} + a_{kj})\ddot{X}_j + b_{kj}\dot{X}_j + CX_j = F_k ,$$

$$k=1, 2\ldots 6, \quad j=1, 2\ldots 6 \quad (17)$$

Where,
M_{kj}=inertia matrix in *k* mode due to the motion in *j* mode
a_{kj}=added mass coefficient matrix of *kj*
b_{kj}=damping coefficient matrix of *kj*
C=hydrostatic restoring force coefficient matrix of *kj*
X_j=vector containing the three translational and three rotational oscillations about the co-ordinate axes in j - mode.

The suffixes *k*, *j*=1, 2, 3, 4, 5, 6 represent surge, sway, heave, roll, pitch and yaw modes, respectively.

3. VALIDATION

To obtain the wave exciting force and motion responses of moored semi submersible, a computer program has been developed. The computation model expected to be validated by the model tests. But since the tests have not been carried out yet, the results obtained from computation of a box (Table 1) for wave exciting forces have been compared with results obtained from WAMIT-MOSES [6]. Motion results have been compared with Sujatha and Soni's [9] results. From these comparisons, it is seen that the surge wave exciting forces are very good agreement with results obtained from WAMIT- MOSES (Fig. 2). On the other hand for roll wave exciting forces, it is seen that higher wave exciting moments are obtained in resonance frequency region (Fig. 3). In the resonance frequency range difficult to compute the forces. Fig.4 shows comparison of pitch motion of another box (Table 2) in reference [9]. In figure 4 characteristic dimension of the structure (*a*) is considered 90m. Overall, a very good agreement has been obtained in all the cases.

Table 1. Principal Particulars of the BOX [6]

L (m)	200
B (m)	40
T (m)	28
Displacement Ton	229640

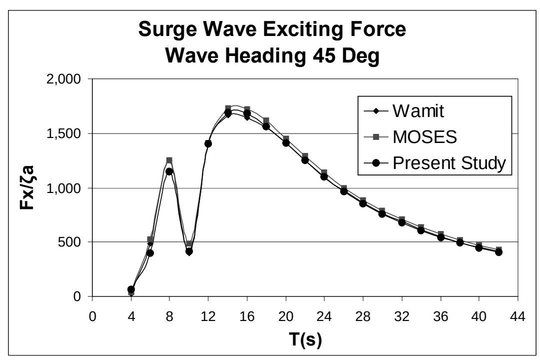

Figure 2: Comparison of Surge wave Exciting force.

Table 2. Principal Particulars of the BOX [9].

L (m)	90
B (m)	90
T (m)	20
Displacement (Ton)	166050
CG(x,y,z) (m)	0, 0, 8.82
Gyration Radii(xx,yy,zz) (m)	37.32, 33.30, 40.08
GM_T=GM_L(m)	14.93

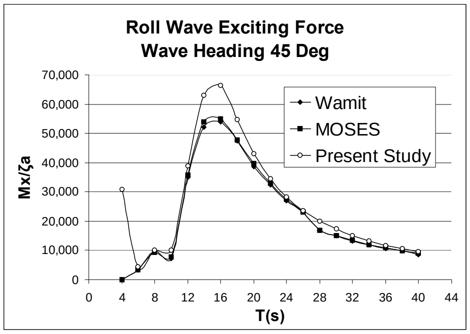

Figure 3: Comparison of Roll wave Exciting Moment.

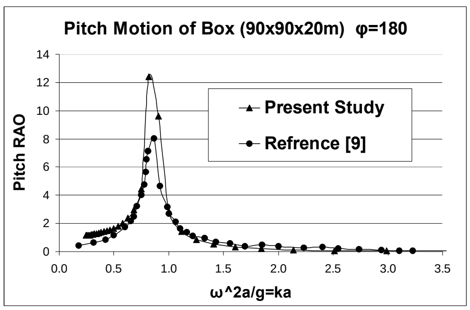

Figure 4: Comparison of Pitch Motion.

4. RESULTS AND DISCUSSION

Computations have been carried for wave exciting forces and motion responses of a semi submersible (Table 3) moored by four connector with stiffness of 206 KN/m (Table 4). In this study the wave exciting forces and motions of a moored semi submersible are plotted against wave period in Figures 5 to Figure 10, at a head sea and water depth of 175 meter. Figure 5, shows non-dimensional surge wave exciting force that drops to 0.02 at 7.4s period, increases dramatically at 11s, then decreases smoothly at 30s. In Figure 6, heave wave exciting force, reaches to 0.27 at 9.2s, falls down rapidly to zero at 15.5s, and then rises slowly to get 0.17 at 30s. Depends on wave period, sometimes wave passes directly through under the structure and excites a little. Wave exciting moment on pitch, Figure 7, decreases slowly from 0.06 at 8.4s, reaches zero at 20s and does not change until 30s.

Figure 8, grows up dramatically and gets peaks 13.8 and 16.4 respectively at 12.2 and 16s and reaches 0.3 at 13.8s. At the other periods surge motion falls down. There is a coupling between pitch RAO (Figure 10) and Surge RAO. This motion peaks at the same periods to get 24.5 and 41. Heave motion has constant amounts of 0.5 until 15s, falls to 0.1 at 15.5s, rises up to 2.7 at 16.5s sharply and falls down to 0.9 to stay constants at the bigger periods.

Table 3. Principal particulars of the Semi submersible.

Pontoon length (m)	66.78
Pontoon depth (m)	6.3
Pontoon beam (m)	13.3
Pontoon centerline separation (m)	45.15
Column longitudinal space (centre)	45.58
Column diameter (m)	10.59
Draft (m)	16.73
Water depth (m)	175
Number of Columns	4

Table 4. Mooring Specification.

No.	x1	y1	z1	x2	y2	z2	Stiffness (KN/m)
1	120	120	6.3	25	29	-14	206
2	120	-120	6.3	25	-29	-14	206
3	-120	-120	6.3	-25	-29	-14	206
4	-120	120	6.3	-25	29	-14	206

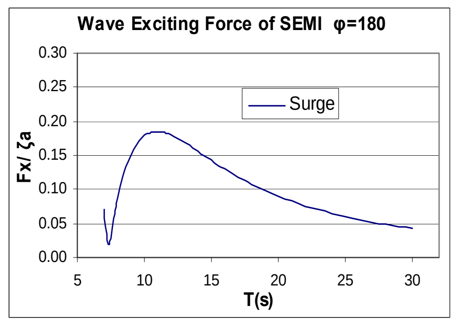

Figure 5: Surge Wave Exciting force

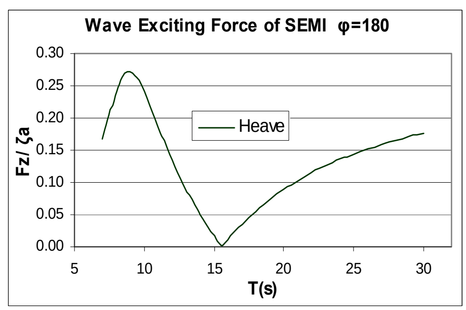

Figure 6: Heave Wave Exciting force

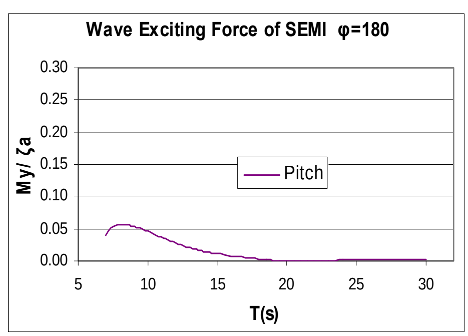

Figure 7: Pitch Wave Exciting Moment

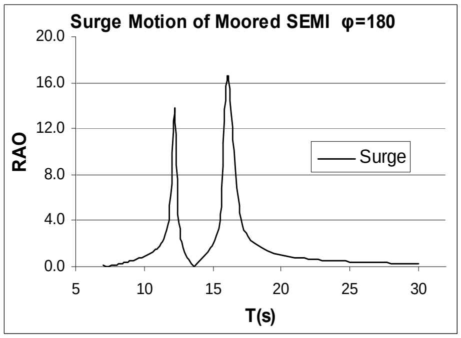

Figure 8: Surge Motion of Moored SEMI

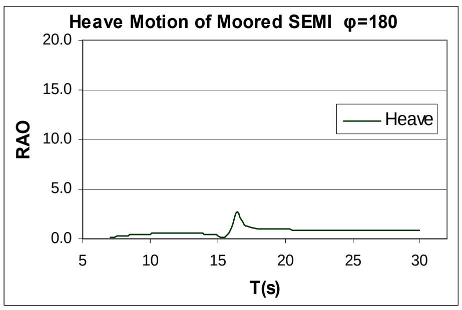

Figure 9: Heave Motion of Moored SEMI

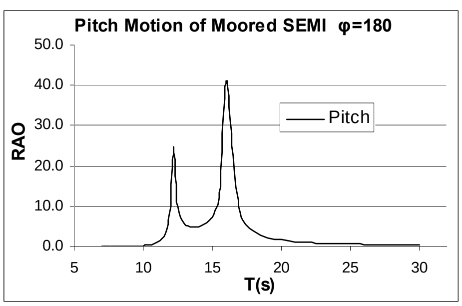

Figure 10: Pitch Motion of Moored SEMI

5. CONCLUSION

A method for and results of computational hydrodynamic studies of wave exciting forces and motion responses of a moored semi submersible have been presented. Wave exciting forces lead to motion of floating/moored structure, which has significant influence on loading and unloading operation. In this paper, the model is validated only with published results but it needs to be validated by model experiment.

Also computations need to be carried out for various depths and different incident angles. However, the program developed for computation of wave exciting forces and motion responses for a freely floating and moored semi submersible numerically expected to be able to predict satisfactorily.

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