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EFFECT OF DIAMETER OF HEATED BLOCK ON MHD FREE CONVECTION FLOW IN A CAVITY

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ABSTRACT

The present work executes a numerical study of magnetohydrodynamic (MHD) free convective flow and heat transfer characteristics inside a square cavity with a uniformly heated solid circular block placed at the centre. The left wall is at a constant temperature and the rest walls of the cavity are considered to be adiabatic. Finite element method based on Galerkin weighted residual approach is used to solve two-dimensional governing mass, momentum and energy equations for steady state, natural convection problem in presence of magnetic field. Results are presented in terms of streamlines, isotherms and average Nusselt number (Nu) at the heated surface for different values of Hartmann number (Ha) and diameter (D) of the block. The results exhibit that the flow structure and the heat transfer rate depend significantly on the mentioned parameters.

Key words: MHD, natural convection, block, cavity, finite element method.

1. INTRODUCTION

Several numerical and experimental methods have been developed to investigate cavities with and without obstacle because these geometries have practical engineering and industrial applications, such as in the design of solar collectors, thermal design of building, air conditioning, cooling of electronic devices, furnaces, lubrication technologies, chemical processing equipment, drying technologies etc. Many authors have recently studied heat transfer in enclosures with partitions, fins and obstacles which influence the convection flow phenomenon. Free convection flow of an electrically conducting fluid in a cavity in the presence of magnetic field is of special technical significance because of its frequent occurrence in many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, thermal insulations and petroleum reservoirs. These types of problems also arise in electronic packages, micro electronic devices during their operations.

House et al. [6] studied the effect of a centered, square, heat conducting body on natural convection in a vertical enclosure. They showed that heat transfer across the cavity enhanced or reduced by a body with a thermal conductivity ratio less or greater than unity. Garandet et al. [4] analyzed the buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. The geometry considered in the numerical study of Oh et al. [9], was a cavity with a heat generating conducting body. Under these situations, it was shown that the flow was driven by a temperature difference across the cavity and a temperature difference caused by the heat-generating source. Roychowdhury et al. [10] analyzed the natural convective flow and heat transfer features for a heated cylinder placed in a square enclosure with different thermal boundary conditions. Natural convection in a horizontal layer of fluid with a periodic array of square cylinder in the interior were conducted by Ha et al. [5], in which they concluded that the transition of the flow from quasi-steady up to unsteady convection depends on the presence of bodies and aspect ratio effect of the cell. Lee et al. [8] considered the problem of natural convection in a horizontal enclosure with a square body. Braga and de Lemos [1] investigated steady laminar natural convection within a square cavity filled with a fixed volume of conducting solid material consisting of either circular or square obstacles. They used finite volume method with a collocated grid to solve governing equations. They found that the average Nusselt number for cylindrical rods was slightly lower than those for square rods. Lee and Ha [7] considered a numerical study of natural convection in a horizontal enclosure with a conducting body. Natural convective heat transfer in square enclosures heated from below was investigated by Calcagni et al. [2]. Sarris et al. [11] studied MHD natural convection in a laterally and volumetrically heated square cavity. They concluded that the usual damping effect of increasing Hartmann number was not found to be straightforward connected with the resulting flow patterns. Roy and Basak [12] analyzed finite element method of natural convection flows in a square cavity with non-uniformly heated wall(s).

In the light of the above literature, it has been pointed out that there is no significant information about MHD free convection processes in an enclosure with a heated obstacle. The present study addresses the effect of size of block and magnetic field which may increase or decrease the heat transfer on natural convection in a square cavity. Numerical solutions are obtained for different diameter of the obstacle and Hartmann number while other parameters such as Rayleigh number and Prandtl number are kept fixed. The numerical results are presented graphically in terms of streamlines and isotherms. Finally the average Nusselt number at the heated surface is calculated.

2. PHYSICAL CONFIGURATION

A schematic diagram of the system considered in the present study is shown in Figure 1. The system consists of a square cavity with sides of length L and a heated circular solid block of diameter d is located at the centre of the enclosure. A Cartesian co-ordinate system is used with origin at the lower left corner of the computational domain. The left wall of the cavity is kept at a constant temperature T_c while the other walls are considered to be adiabatic. The uniform temperature of the obstacle is assumed to be T_h. Here T_c is less than T_h. A magnetic field of strength B_0 is applied horizontally normal to the side walls.

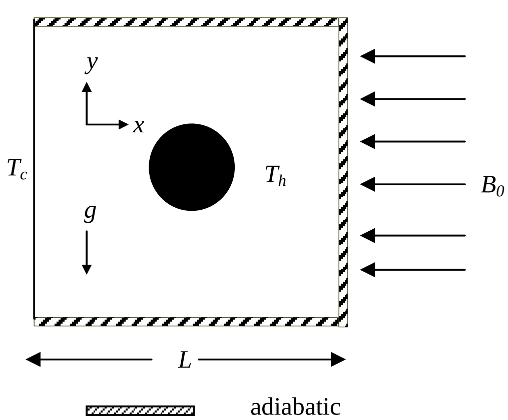


Figure 1. Schematic diagram of the problem

3. MATHEMATICAL FORMULATION

In the present problem, it can be considered that the flow is steady, two-dimensional, laminar incompressible and there is no viscous dissipation**.** The gravitational force (g) acts in the vertically downward direction and radiation effect is neglected. The governing equations under Boussinesq approximation are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) + g\rho\beta(T-T_c)-\sigma B_0^2\, v \quad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

where $\alpha = k/\rho C_p$ is the thermal diffusivity of the fluid. The boundary conditions are

$$u(x,0)=u(x,L)=u(0,y)=u(L,y)=0$$
$$v(x,0)=v(x,L)=v(0,y)=v(L,y)=0$$
$$T(0,y)=T_c$$
$$\frac{\partial T(x,0)}{\partial y}=\frac{\partial T(x,L)}{\partial y}=\frac{\partial T(L,y)}{\partial x}=0$$

At the circular body surface $u(x,y)=v(x,y)=0$, $T(x,y)=T_h$

The local Nusselt number at the heated circular body in the square enclosure is evaluated by the following expression in dimensional form as

$Nu_{local}=hd/k$, where h is the convective heat transfer coefficient.

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ D=\frac{d}{L},\ U=\frac{uL}{\alpha},\ V=\frac{vL}{\alpha},$$
$$P=\frac{pL^2}{\rho\alpha^2},\ \theta=\frac{T-T_c}{T_h-T_c}$$

After substitution of the above variables, equations (1) to (4) transformed into the following non-dimensional equations

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)$$
$$+Ra\,Pr\,\theta-Ha^2\;Pr\,V$$
$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)$$

where $Pr=\frac{\nu}{\alpha}$ is Prandtl number, $Ra=\frac{g\beta(T_h-T_c)L^3}{\nu\alpha}$ is Rayleigh number and Ha is Hartmann number which is defined as $Ha^2=\frac{\sigma B_0^2\,L^3}{\mu}$.

The corresponding boundary conditions then take the following form

At the inside and on the wall of enclosure fluid pressure $P = 0$

$$U(X,\,0)=U(X,\,1)=U(0,\,Y)=U(1,\,Y)=0$$
$$V(X,\,0)=V(X,\,1)=V(0,\,Y)=V(1,\,Y)=0$$
$$\theta(0,\,Y)=0$$
$$\frac{\partial\theta(X,\,0)}{\partial Y}=\frac{\partial\theta(X,\,1)}{\partial Y}=\frac{\partial\theta(1,\,Y)}{\partial X}=0$$

At the circular body surface
$$U(X,Y)=V(X,Y)=0,\quad \theta(X,Y)=1$$

The average Nusselt number at the body of the enclosure may be expressed as

$$Nu=-\int_0^{L_s/L}\frac{\partial\theta}{\partial n}\,ds$$

where $\frac{\partial\theta}{\partial n}=\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}$ and s is the coordinate along the circular surface.

4. NUMERICAL TECHNIQUE

The numerical procedure used in this work is based on the Galerkin weighted residual method of finite element formulation. The application of this technique is well described by Taylor and Hood [13] and Dechaumphai [3]. In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method.

4.1 Grid Refinement Check

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for *Ha* = 50, *Pr* = 0.7, *Ra* = 10^5 and *D* = 0.25. The extreme value of *Nu* is used as a sensitivity measure of the accuracy of the solution and is selected as the monitoring variable. Considering both the accuracy of numerical values and computational time, the present calculations are performed with 39284 nodes and 5936 elements grid system.

Table 1. Grid Sensitivity Check at *Ha* = 50, *Pr* = 0.7, *Ra* = 10^5 and *D* = 0.25.

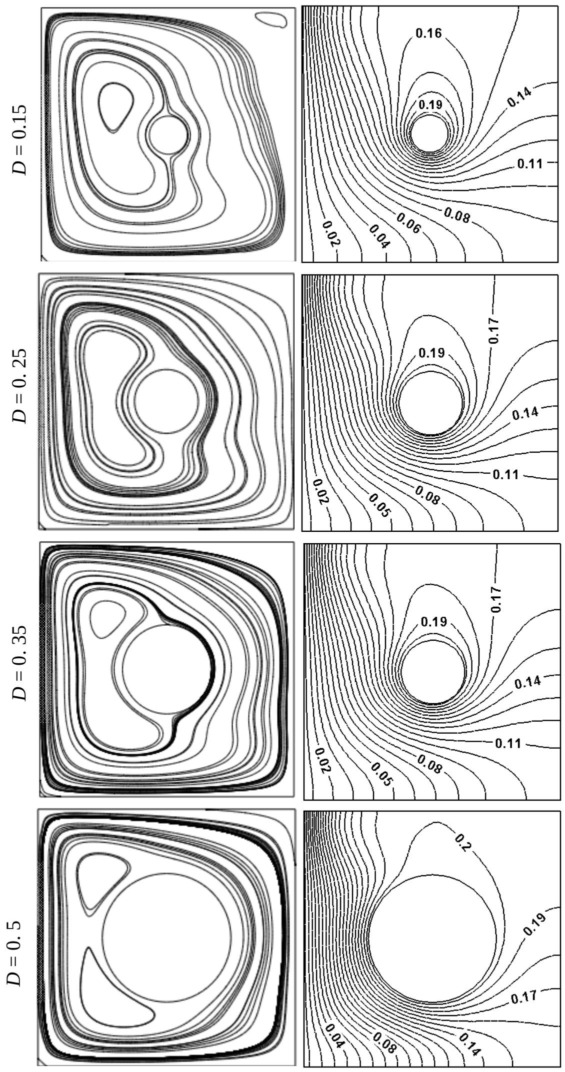
Nodes (elements)	7432 (1096)	11988 (1784)	26536 (3992)	39284 (5936)	79500 (12080)
Nu	0.578491	0.596590	0.598085	0.598592	0.598592
Time (s)	226.265	292.594	388.157	421.328	627.375

5. RESULT AND DISCUSSION

Effect of the physical parameter (*D*) and Hartmann number (*Ha*) on heat transfer and fluid flow inside the cavity has been analyzed in the present study. Heat transfer rate in terms of the average Nusselt number at the heated block in the cavity has been also discussed. The ranges of *D* and *Ha* for this investigation vary from 0.15 to 0.5 and 0 to 50 respectively whereas other parameters are fixed at *Ra* = 10^5 and *Pr* = 0.7.

The influence of diameter *D* for a fixed *Ha* (= 50) on the flow field is depicted in Fig 2 (a). The flow with *D* = 0.15 has been affected by the buoyancy force, thus creating two rotating cells. The right top one is small which vanishes with increasing *D*. The left one covers almost the whole cavity including the block. It contains a small vortex. The size of this vortex increases due to the increasing values of *D*. For *D* = 0.50, the existing recirculation region becomes larger and two inner vortices are developed. Fig. 2 (b) illustrates the temperature field in the flow region. The stratified isothermal lines concentrated near the left wall and the heated body. The concentrated temperature region near the block becomes denser for larger *D* and consequently the isothermal lines almost disappear from the right top portion of the cavity.

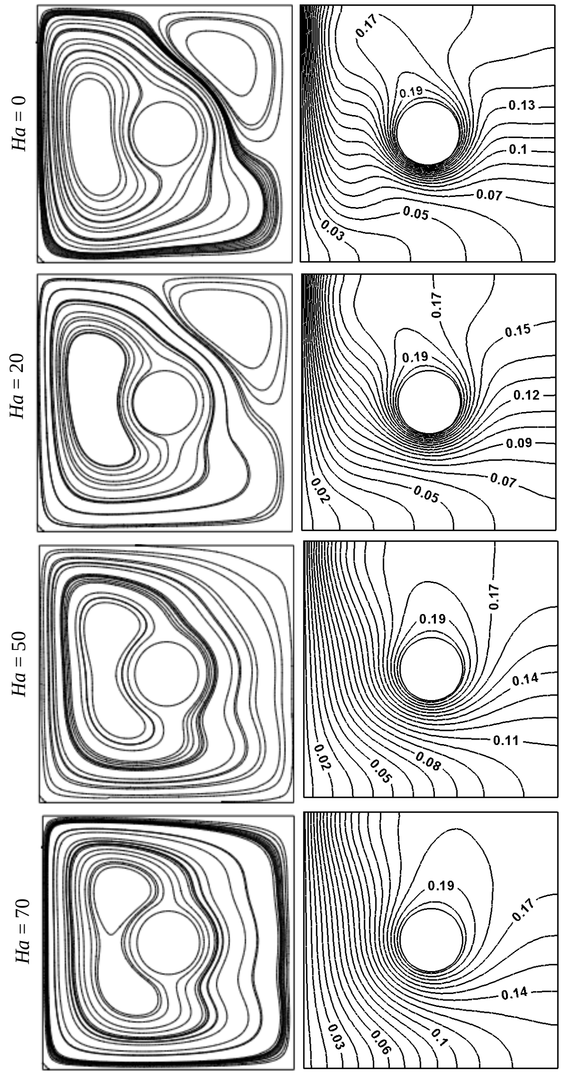
The influence of Hartmann number *Ha* (from 0 to 70) on streamlines as well as isotherms for the present configuration at *Ra* = 100000, *Pr* = 0.7, *D* = 0.25 has been demonstrated in Fig. 3. The flow with *Ha* = 0 creates two vortices near both side of the heated body due to the buoyancy force. The vortex on the right top corner losses its strength for higher *Ha* and finally disappeared. The corresponding isotherm patterns are shown in Fig. 3 (b). The high temperature region is concentrated near the circular obstacle for *Ha* = 0. The lines become less bend from the left top corner because of increasing values of *Ha.* The concentrated temperature region near the heated surface becomes thin for larger *Ha.*



(a) (b)

Fig. 2: (a) Streamlines and (b) Isotherms for varying of *D* with *Ra* = 100000, *Ha* = 50 and *Pr* = 0.7

In order to evaluate how the presence of magnetic field and diameter of the obstacle affect the heat transfer rate along the heated surface, the average Nusselt number is plotted as a function of Hartmann



(a) (b)

Fig. 3: (a) Streamlines and (b) Isotherms for varying of *Ha* with *Ra* = 10^5, *D* = 0.25 and *Pr* = 0.7

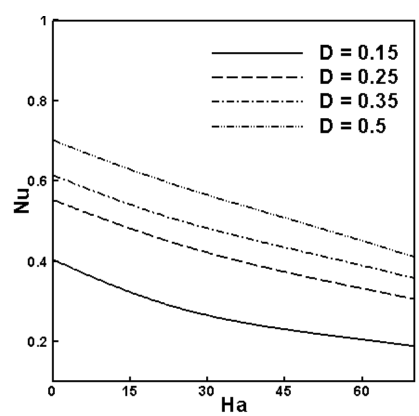


Fig. 4: Effect of *Ha* and *D* on *Nu* while *Ra* = 10^5 and *Pr* = 0.7

number as shown in Fig. 4 for four different diameter (D = 0.15, 0.25, 0.35 and 0.5) while $Ra = 10^5$ and Pr = 0.7. It is observed that Nu decreases with the increasing Ha and increases with the growing D. The maximum heat transfer rate is obtained for the lowest Ha. This is because the magnetic field retards the flow. Also the average Nusselt number is found highest for the largest value of D because larger body having larger surface is capable to transfer more heat.

6. CONCLUSION

A finite element method is used to make the present investigation for steady-state, incompressible, MHD free convection flow in a cavity with a heated body. The major conclusions have been drawn as follows:

The diameter of the body has a significant effect on the flow and temperature fields. Buoyancy-induced vortex in the streamlines increased and thermal layer between the left wall and the heated surface become denser for increasing values of D.

The influence of Magnetic parameter Ha on streamlines and isotherms are remarkable. The vortex in the streamlines decreased and thermal layer near the heated surface becomes thin with increasing values of Ha.

The average Nusselt number Nu at the heated surface increases for larger values of Ra and D where as it decreases with the increasing values of Ha.

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