Proceedings of MARTEC 2010
The International Conference on Marine Technology
11-12 December 2010, BUET, Dhaka, Bangladesh

MAX-PLUS APPROACH TO SCHEDULING PROBLEMS OF BLOCK ASSEMBLY LINES

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ABSTRACT

The paper shows pull-type scheduling plans of block assembly lines in a shipyard based on max-plus approach. The shipyard is assumed to have assembly lines for large-scale/medium-scale blocks as conveyer lines and also have stockyards for storing blocks. The dynamics of assembly lines and stockyards can be mathematically modelled in the form of linear system representations using max-plus algebra. This makes it possible to obtain pull-type scheduling plans by solving model predictive equations. Compared with conventional scheduling plans, max-plus scheduling plans indicate that the capacity overflow in stockyards can be avoided and also the workplace limitation on conveyer lines can be observed strictly under the consideration on assembly order of blocks.

Key words: shipyard scheduling, max-plus algebra, block assembly, conveyer line, stockyard.

1. INTRODUCTION

In recent years, the global flow of goods into BRIC (Brazil, Russia, India, and China) countries has been accelerated. There are strong demands from ship owners on bulk carriers to transport unpackaged bulk cargo such as cereals, coal, ore, and cement. In Japan, several shipbuilding companies are trying to improve the productivity of bulk carriers. In particular, Oshima Shipbuilding is a shipbuilding company that specializes in the fabrication of bulk carriers, especially dry cargo carriers. An air photo of the shipyard is shown in Photo.1. Although the company has been delivering annually around 30 ships, the more productivity is expected. For the purpose, the more elaborate or systematic scheduling approach is required, that is, the assembly start times of several hundred blocks in each production cycle must be determined to meet the erection dates just in time and not to cause the overflow in stockyards. It is not so easy to solve such a scheduling problem using commercial software packages.

In our previous papers[1],[2], we have shown that shipbuilding lines consisting of block assembly conveyers and block stockyards can be mathematically modelled as state-space representations using max-plus algebra[3], which are scheduled based on model predictive control theory. The scheduling problem is concerned with how to change the push type to the pull type. In particular, in order to solve the actual problem, it is crucial to develop functions of conveyer model and stock model with functions of serial and parallel connections using Max-Plus Toolbox on MATLABTM [4] or VBA in EXCEL.

The paper follows up the same research direction to demonstrate that max-plus scheduling plans of 28 ships produced in 2009 to 2010 are effectively obtained. In the chapter 2, some prerequisites are given. Pull-type scheduling results for assembly lines for large-scale and medium-scale blocks are shown in chapter 3 and chapter 4 respectively.

Photo. 1: Oshima Shipbuilding in Japan

2. PREREQUISITES

2.1 Max-Plus Algebra Based Linear System Representation [1]

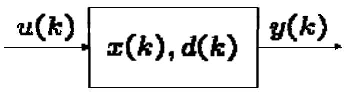

Figure 1. Production System

All Consider a production system for the k-th product as shown in Fig.1, where $x(k)$ is the start time, $d(k)$ is the process time, $u(k)$ is the arrival time of parts/ materials, and $y(k)$ is the stop time. Then, the following relation holds:

$$\begin{cases} x(k) = \max\{y(k-1), u(k)\} \\ y(k) = x(k) + d(k) \end{cases}. \quad (1)$$

The first equation shows that the start time is determined as the last stop time or the arrival time of parts, whichever is later. The second equation is a trivial relation between the start time and the stop time. Replacing the max operation and the plus operation in (1) by $\oplus$ and $\otimes$ respectively brings

$$\begin{cases} x(k) = d(k-1) \otimes x(k-1) \oplus u(k) \\ y(k) = d(k) \otimes x(k) \end{cases}. \quad (2)$$

Letting $A(k-1) = d(k-1), B(k) = e, C(k) = d(k)$, and omitting $\otimes$, (2) is written as

$$\begin{cases} x(k) = A(k-1)x(k-1) \oplus B(k)u(k) \\ y(k) = C(k)x(k) \end{cases}. \quad (3)$$

This is called as a linear system representation based on max-plus algebra. The first equation is the state equation and the second equation is the output equation. In general, x, u, y are vectors of appropriate dimensions and A, B, C are matrices of compatible dimensions.

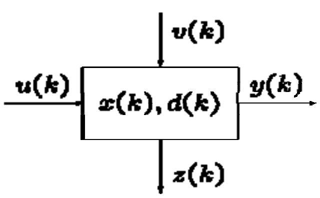

Figure 2. Production System of 2-Port Type

Consider the system with 2 kinds of inputs and outputs as shown in Fig.2. In addition to $u(k), y(k)$ as defined in the above, $v(k)$ is the available time of a machine, $z(k)$ is the released time of a machine.

$$\begin{cases} x(k+1) = A(k-1)x(k-1) \oplus \underbrace{[B_1(k) \quad B_2(k)]}_{B(k)} \begin{bmatrix} u(k) \\ v(k) \end{bmatrix} \\ \begin{bmatrix} y(k) \\ z(k) \end{bmatrix} = \underbrace{\begin{bmatrix} C_1(k) \\ C_2(k) \end{bmatrix}}_{C(k)} x(k) \oplus \underbrace{\begin{bmatrix} D_{11}(k) & D_{12}(k) \\ D_{21}(k) & D_{22}(k) \end{bmatrix}}_{D(k)} \begin{bmatrix} u(k) \\ v(k) \end{bmatrix} \end{cases}. \quad (4)$$

It is a key to distinguish the production cycle from the machine utilization cycle in order to take into account of resource constraints on capacities or workplaces.

2.2 Conveyer Model [2]

A conveyer line is useful to realize CONWIP (constant works in process), in which m workplaces are moved with a constant pitch c. Fig.3 shows such a conveyer mechanism, which is successfully modelled as a linear system representation in the following.

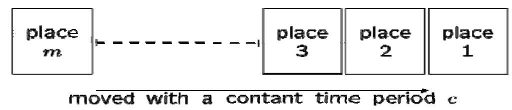

Figure 3. Schematic Diagram of Conveyer Line

For the *k*-th product, let n be the number of parts which are fabricated in the conveyer line consisting of m work-places. Assume that the last r' parts for the (*k*-1)-th product occupy the beginning r' workplaces, and also that the last *r* parts for the *k*-th product occupy the beginning *r* workplaces. We consider the following four cases.

- **Case1** $n \geq m$ $(s = m - r'. r = n - s - qm. q \geq 1)$
- **Case2** $n \geq m$ $(s = m - r'. r = n - s < m \ (q = 0) \Rightarrow r + s = n \geq m = r' + s \Rightarrow r \geq r')$
- **Case3** $n < m$ $(s = m - r'. r = n - s \geq 0 \Rightarrow r + s = n < m = r' + s \Rightarrow r < r')$
- **Case4** $n < m$ $(s = m - r'. n < s. r = r' + n)$

The paper[2] derives the results shown in Fig.4, 5 and 6 corresponding to Case 1, 3 and 4 respectively.

$$\begin{cases} x(k) = \underbrace{\begin{bmatrix} B_1(k) & B_2(k) \begin{bmatrix} \epsilon_{m-r' \times r'} & \epsilon_{m-r' \times m-r'} \\ e_{r' \times r'} & \epsilon_{r' \times m-r'} \\ \epsilon_{n-m \times r'} & \epsilon_{n-m \times m-r'} \end{bmatrix} \end{bmatrix}}_{B} \begin{bmatrix} u(k) \\ \bar{v}(k) \end{bmatrix} \\ \begin{bmatrix} y(k) \\ \bar{z}(k) \end{bmatrix} = \underbrace{\begin{bmatrix} C_1(k) \\ \begin{bmatrix} \begin{bmatrix} \epsilon_{r \times n-r} & \epsilon_{r \times r} \end{bmatrix} C_1(k) \\ \begin{bmatrix} \epsilon_{1 \times n-m} & e & \epsilon_{1 \times m-1} \end{bmatrix} C_1(k) \oplus \begin{bmatrix} \epsilon_{1 \times n-1} & e \end{bmatrix} C_2 \\ \begin{bmatrix} \epsilon_{m-r-1 \times n-m+1} & \epsilon_{m-r-1 \times m-r-1} & \epsilon_{m-r-1 \times r} \end{bmatrix} C_1(k) \end{bmatrix} \end{bmatrix}}_{C} x(k) \end{cases}$$

Figure 4. Max-plus Model of Conveyer Line (Case1)

$$\begin{cases} x(k) = \underbrace{\begin{bmatrix} B_1(k) & B_2(k) \begin{bmatrix} \epsilon_{m-r' \times r} & \epsilon_{m-r' \times m-n} & e_{m-r' \times m-r'} \\ e_{r \times r} & \epsilon_{r \times m-n} & \epsilon_{r \times m-r'} \end{bmatrix} \end{bmatrix}}_{B} \begin{bmatrix} u(k) \\ \bar{v}(k) \end{bmatrix} \\ \begin{bmatrix} y(k) \\ \bar{z}(k) \end{bmatrix} = \underbrace{\begin{bmatrix} C_1(k) \\ \begin{bmatrix} \begin{bmatrix} \epsilon_{r \times n-r} & e_{r \times r} \end{bmatrix} C_1(k) \\ \begin{bmatrix} \epsilon_{1 \times n-1} & e \end{bmatrix} C_2 \\ \epsilon_{m-n-1 \times n} \\ \begin{bmatrix} \epsilon_{m-r' \times n-m} & e_{m-r' \times m-r'} & \epsilon_{m-r' \times r'} \end{bmatrix} C_1(k) \end{bmatrix} \end{bmatrix}}_{C} x(k) \\ + \underbrace{\begin{bmatrix} \epsilon_{n \times n} & \begin{bmatrix} \epsilon_{n \times r} & \epsilon_{n \times m-n} & \epsilon_{n \times r'} \end{bmatrix} \\ \begin{bmatrix} \epsilon_{r \times n} \\ \epsilon_{m-n \times n} \\ \epsilon_{m-r' \times n} \end{bmatrix} & \begin{bmatrix} \epsilon_{r \times r} & \epsilon_{r \times m-n} & \epsilon_{r \times m-r'} \\ \epsilon_{m-n \times r} & e_{m-n \times m-n} & \epsilon_{m-n \times m-r'} \\ \epsilon_{m-r' \times r} & \epsilon_{m-r' \times m-n} & \epsilon_{m-r' \times m-r'} \end{bmatrix} \end{bmatrix}}_{D} \begin{bmatrix} u(k) \\ \bar{v}(k) \end{bmatrix} \end{cases}$$

Figure 5. Max-plus Model of Conveyer Line (Case3)

$$\begin{cases} x(k) = \underbrace{\begin{bmatrix} B_1(k) & B_2(k) \begin{bmatrix} \epsilon_{n \times r'} & e_{n \times n} & \epsilon_{n \times m-r} \end{bmatrix} \end{bmatrix}}_{B} \begin{bmatrix} u(k) \\ \bar{v}(k) \end{bmatrix} \\ \begin{bmatrix} y(k) \\ \bar{z}(k) \end{bmatrix} = \underbrace{\begin{bmatrix} C_1(k) \\ \begin{bmatrix} \epsilon_{r' \times n} \\ C_1(k) \\ \begin{bmatrix} \epsilon_{1 \times n-1} & e \end{bmatrix} C_2 \\ \epsilon_{m-r-1 \times n} \end{bmatrix} \end{bmatrix}}_{C} x(k) \\ + \underbrace{\begin{bmatrix} \epsilon_{n \times n} & \begin{bmatrix} \epsilon_{n \times r'} & \epsilon_{n \times n} & \epsilon_{n \times m-r} \end{bmatrix} \\ \begin{bmatrix} \epsilon_{r' \times n} \\ \epsilon_{n \times n} \\ \epsilon_{m-r \times n} \end{bmatrix} & \begin{bmatrix} e_{r' \times r'} & \epsilon_{r' \times n} & \epsilon_{r' \times m-r} \\ \epsilon_{n \times r'} & \epsilon_{n \times n} & \epsilon_{n \times m-r} \\ \epsilon_{m-r \times r'} & \epsilon_{m-r \times n} & e_{m-r \times m-r} \end{bmatrix} \end{bmatrix}}_{D} \begin{bmatrix} u(k) \\ \bar{v}(k) \end{bmatrix} \end{cases}$$

Figure 6. Max-plus Model of Conveyer Line(Case4)

2.3 Scheduling by Solving Model Predictive Equation[5]

Assume that a linear system representation of production system to be scheduled is obtained as (3). From the state equation, the following relation is derived.

$$\begin{cases} x(k) = A(k-1)x(k-1) \oplus B(k)u(k) \\ x(k+1) = A(k)x(k) \oplus B(k+1)u(k+1) \\ \quad = A(k)A(k-1)x(k-1) \\ \quad \oplus A(k)B(k)u(k) \oplus B(k+1)u(k+1) \\ \quad \vdots \end{cases} . \quad (5)$$

Using the output equation, (5) becomes

$$\underbrace{\begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \end{bmatrix}}_{Y(k)} = \Gamma(k)x(k-1) \oplus \Delta(k) \underbrace{\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \end{bmatrix}}_{U(k)}, \quad (6)$$

where in the case of 1-step-ahead prediction,

$$\begin{cases} \Gamma(k) = C(k)A(k-1) \\ \Delta(k) = C(k)B(k) \end{cases}, \quad (7)$$

in the case of 2-step-ahead prediction,

$$\begin{cases} \Gamma(k) = \begin{bmatrix} C(k)A(k-1) \\ C(k+1)A(k)A(k-1) \end{bmatrix} \\ \Delta(k) = \begin{bmatrix} C(k)B(k) & \varepsilon \\ C(k+1)A(k)B(k) & C(k+1)B(k+1) \end{bmatrix} \end{cases}. \quad (8)$$

Now let the delivery times be

$$R(k) = \begin{bmatrix} r(k) \\ r(k+1) \\ \vdots \end{bmatrix}. \quad (9)$$

It is required to determine the arrival times of parts $U(k)$ satisfying

$$R(k) = \Gamma(k)x(k-1) \oplus \Delta(k)U(k). \quad (10)$$

In the case that the following condition holds,

$$R(k) \geq \Gamma(k)x(k-1), \quad (11)$$

(10) becomes

$$R(k) = \Delta(k)U(k). \quad (12)$$

which does not have the solution $U(k)$ in general. Therefore, it is required to obtain the maximal solution $U(k)$ of (12) satisfying

$$\Delta(k)U(k) \leq R(k). \quad (13)$$

In the case that (11) does not hold, it is impossible to obtain the appropriate $U(k)$. Then it is necessary to change (6) itself by reducing the processing times.

3. SCHEDULING OF LARGE-SCALE BLOCK ASSEMBLY LINES

The half of the shipyard layout considered in the paper is depicted in Fig.7. There is a dock which can include four ships. One pair of Ship A/ Ship B and another pair of Ship C/ Ship D are constructed alternately. In order to manage such a dock cycle, it is necessary to prepare all the blocks until the erections begin. Because of the short period of erection, the shipyard has Stockyard #1/ Stockyard #2 to store blocks of two ships, that is, 120 blocks. Also it has 8 block assembly lines as conveyers. The related parameters are shown in Table1. Each line is able to fabricate blocks with a constant pitch.

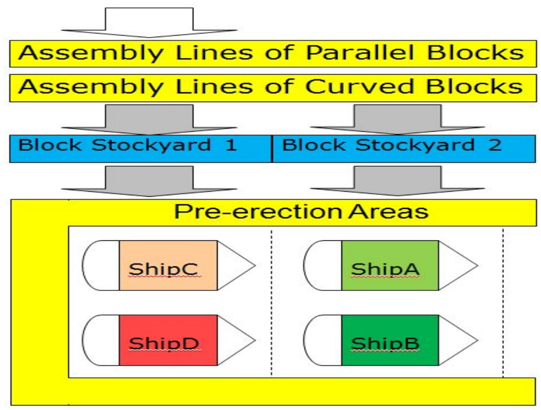

Figure 7. Block Assembly Lines and Stockyards

Table1. Parameters on Block Assembly Lines

line#	*m*	*c*	*m'*	line#	*m*	*c*	*m'*
1	6	7/8	21	5	4	6/4	10
2	1	7/8	4	6	9	5/9	22
3	1	7/8	4	7	7	5/7	15
4	10	7/10	34	8	4	4/4	10

m: number of workplaces on conveyer
c: pitch of conveyer
m': number of stock places assigned to conveyer

Fig.8 shows the actual numbers of large-scale blocks to be fabricated in 2009 to 2010. There are 7 cycles in which each cycle has around 220 blocks for four ships, that is, around 60 blocks for one ship.

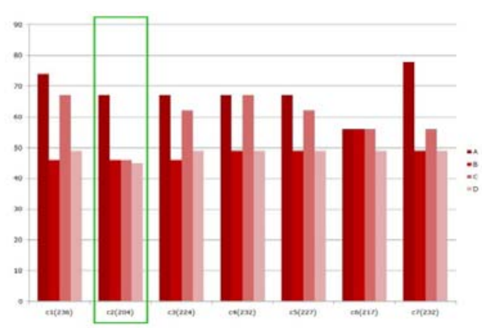

Figure 8. Numbers of Blocks Fabricated in 2009 to 2010

For example, the conventional scheduling plan for Ship C in the Cycle 2 is shown in Fig.9. As for the other Ship A,B,D in the other Cycle 1,3,4,5,6,7, we have the same figures. The red bars indicate the election dates for Ship C. Also the yellow bars, the blue bars and the green bars indicate pre-election periods, stock periods and assembly periods respectively. We can reveal the idling days by taking minimum stock periods which is necessary for pre-fitting and painting as in Fig.10. Our scheduling problem is to reduce the idling days as many as possible such that the facility constraints on conveyers and stockyards are satisfied as in Fig.11.

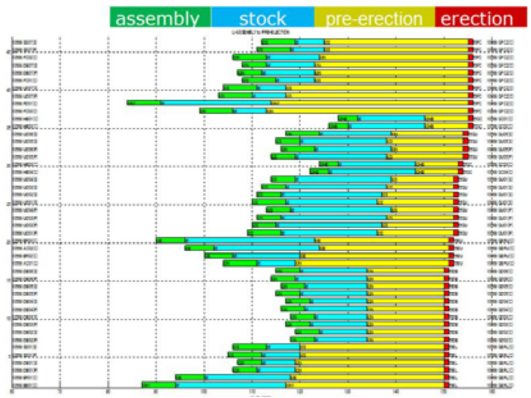

Figure 9. Conventional Scheduling for Ship C

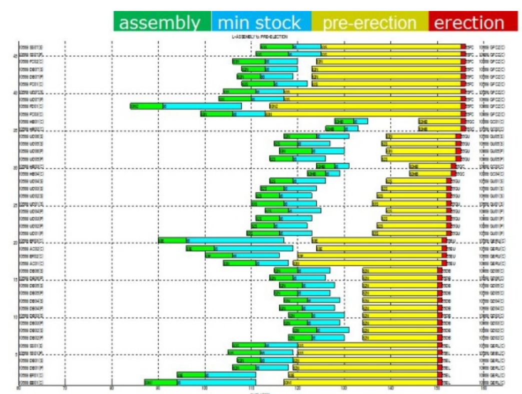

Figure 10. Revealing Idling Days in Fig.9

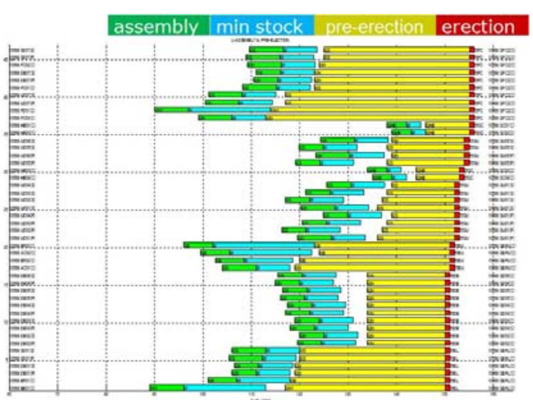

Figure 11. Max-plus Scheduling for Ship C

The scheduling plan for Line #1 is made as follows. So are the other lines. At first, consider all the blocks for Ships A,B,C,D fabricated in Line #1 in Fig.12-LHS (Left-Hand Side). Then sort them according to the dates given by subtracting the minimum stock periods from the pre-erection dates as shown in Fig.13-LHS. Then make the sorted scheduling plan based on the max-plus method in Sec.2.3 as in Fig.13-RHS in which the equation (12) with the parameters (7) of 1-step-ahead prediction is solved. Finally obtain the max-plus scheduling plan in Fig.12-RHS by sorting reversely. Note that the idling days are reduced from 58 to 14.8.

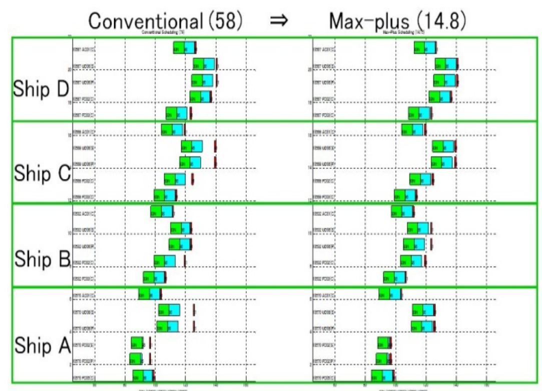

Figure 12. Scheduling for Line#1 in Cycle 2

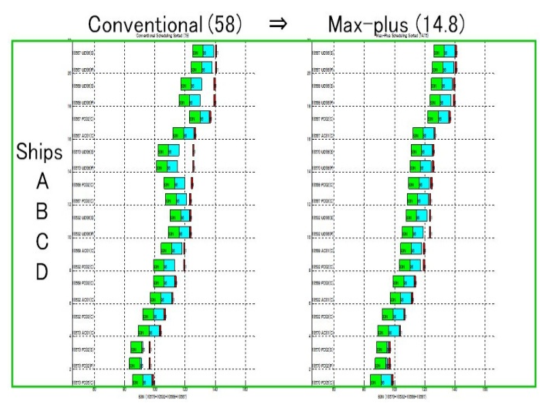

Figure 13. Sorted Scheduling for Line#1 in Cycle 2

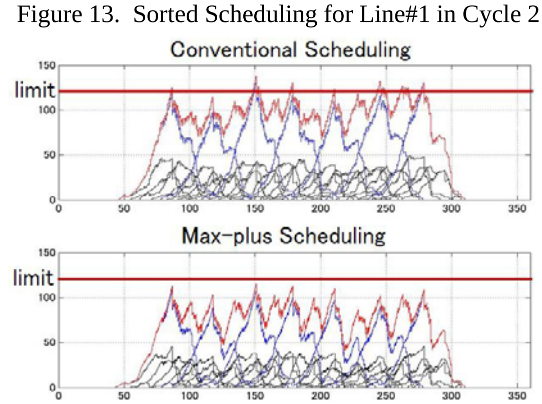

Figure 14. Counting Blocks Stored in Stockyards

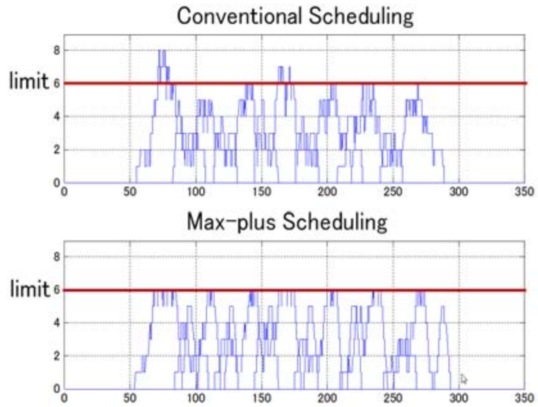

Figure 15. Counting Workplaces Occupied in Line#1

In order to check that the facility constraints on conveyers and stockyards are satisfied, Fig.14 and Fig.15 are presented. Fig.14 shows the counting results of blocks stored in stockyards, and Fig.15 shows the counting results of workplaces occupied in Line#1. Although there are some violations in the conventional scheduling, the max-plus scheduling have no violation.

4. SCHEDULING OF MEDIUM-SCALE BLOCK ASSEMBLY LINES

The another half of the shipyard layout considered in the paper is depicted in Fig.16. The large-scale blocks are made from several medium-scale sub-blocks which are fabricated in 5 sub-block assembly lines as conveyers. The related parameters are shown in Table2.

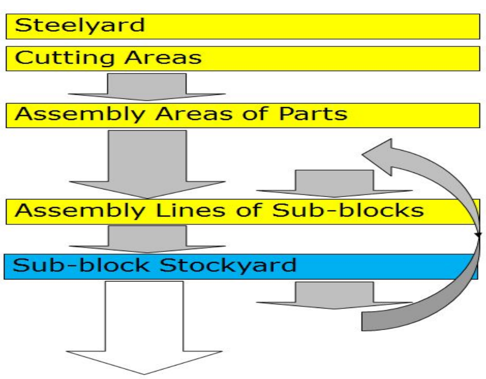

Figure 16. Sub-block Assembly Lines & Stockyards

Table2. Parameters on Sub-block Assembly Lines

line#	*m*	*c*	*m'*
11	8	4/8	10
12	24	7/24	31
13*	10	4/10	13
14	9	4/9	12
15*	10	4/10	14
* lines with feedback flows			

In Table 2, note that Line #13 and #15 are of feedback type. This means that some sub-blocks are utilized to fabricate the other sub-blocks. Specifically, sub-blocks in Line #13 and #15 are fed back to Line #15. The scheduling plans for these lines are made as follows. At first, they are planned for sub-blocks without feedback as shown in Fig.17 and Fig.18, in which the target dates are taken as the start dates of the corresponding large-scale blocks. From these results, the target dates for sub-blocks fed back in Line #13 and #15 are determined. They must be pushed in scheduling on Line #15 as shown in Fig.19.

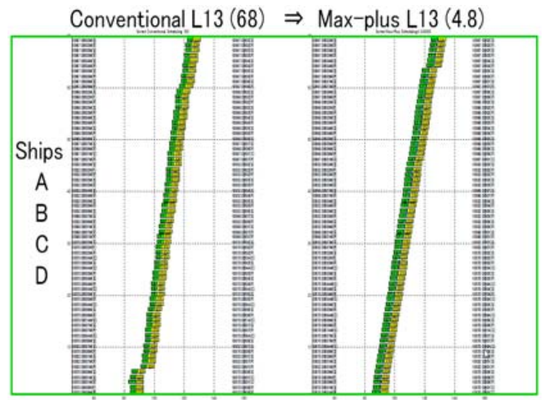

Figure 17. Sorted Pre-Scheduling for Line#13

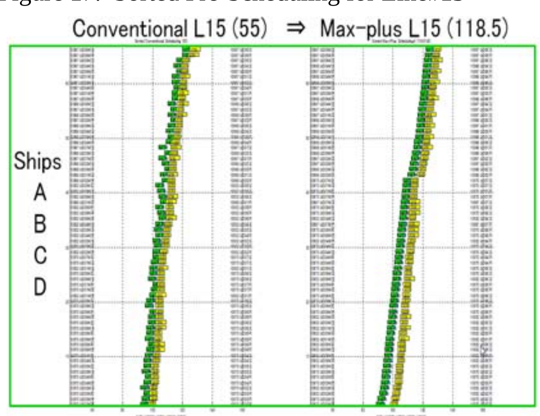

Figure 18. Sorted Pre-Scheduling for Line#15

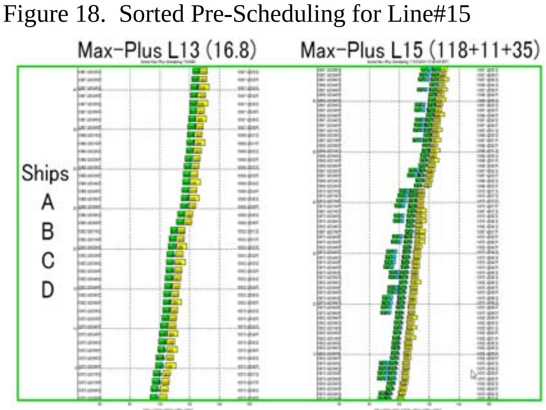

Figure 19. Sorted Scheduling for Line#13 and #15

The sorting is done for all sub-blocks fabricated in Line #13 and #15 according to the dates given by subtracting the minimum stock periods from their target dates. Fig 20 shows the conventional scheduling plans for Line#13 and #15, where the numbers of workplaces occupied in Line #13 and #15 are larger than the limitations. Fig.21 shows the max-plus scheduling plans for Line#13 and #15, where the idling days are increased compared with the conventional scheduling ones, but the counting results of sub-blocks stored in stockyards are not so deteriorated as shown in Fig.22.

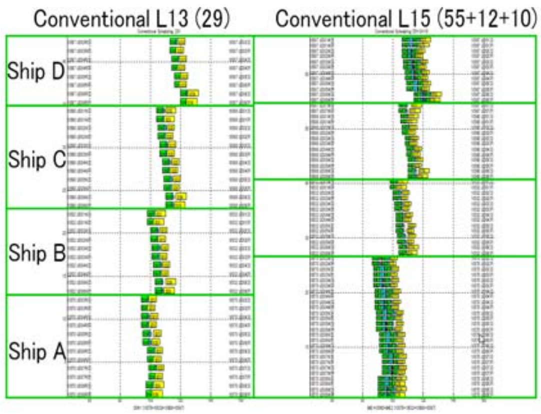

Figure 20. Conventional Scheduling for Line#13, #15

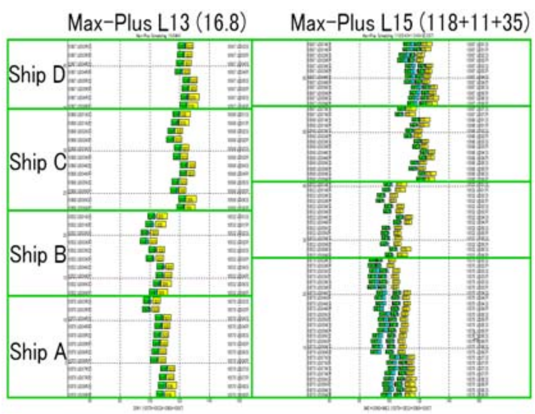

Figure 21. Max-plus Scheduling for Line#13 and #15

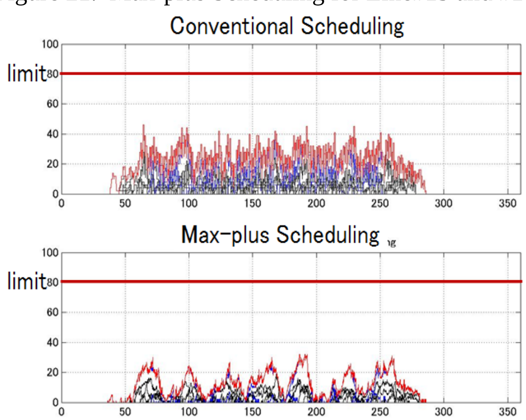

Fig.22. Counting Sub-Blocks Stored in Stockyard

5. CONCLUSION

The contributions of the research at present are summarized as follows:

1. pull-type scheduling methodology of block assembly lines and stockyards based on conveyer/ stock models to satisfy facility constraints and the *slack* dispatching rule to determine the order of blocks fabricated.
2. pull-type scheduling methodology of sub-block assembly lines with the feedback flow, that is, some sub-blocks are utilized to fabricate the other sub-blocks

The further research will be concerned with a multi-cycle scheduling problem which is necessary for discussing a overlapping problem between two cycles, and workload balancing problems[6]

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