NUMERICAL INVESTIGATION OF INTERNAL HEAT GENERATION AND ABSORPTION ON MAGNETOHYDRODYNAMIC MIXED CONVECTION IN AN ENCLOSURE WITH SINUSOIDAL BOTTOM WALL

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Abstract

Influence of internal heat absorption and generation on magnetohydrodynamic mixed convection inside an enclosure is numerically investigated. The cavity used for flow and heat transfer is bounded by adiabatic upper wall, cold perpendicular walls and sinusoidal bottom wall. Finite element process is used to resolve the developed governing equation of the physical model. Flow pattern and rate of heat transfer owing to the variation of parameters like as Hartmann number, Richardson number and influence of inner heat generation and absorption will be studied in details. Our numerical results show that the heat transfer rate increase with the augmentation of heat generation parameter and Richardson number. The results are validated against with the previous published work.

Keywords: Magnetic Field, Mixed Convection, Sinusoidal, Finite Element Formulation, Heat Generation and Absorption.

1. INTRODUCTION

The impact of a magnetic field on the combined convective fluid flow with electrical conductivity in an enclosure has drawn significant concentration because of its vast range of applications in geothermal reservoirs, cooling of nuclear reactors, thermal insulations petroleum reservoirs, electronic packages, etc. Many experimental and numeric analyses on lid-driven enclosure flow and heat transmission throughout the last several decades. Ismael et al. [1] brought out a numerical simulation of combined convection inside a lid-driven four-sided chamber with partial slip. Saha et al. [2] scrutinized mixed convection heat metastasis inside a lid-driven enclosure with a wavy bottom surface. Abu-Nanda and Chamkha [3] studied the mixed convective discharge of a nanofluid inside a lid-driven enclosure, including an undulating wall. Asad et al. [4] examined the impact of a rectangular heat source in a confined room on free convective flow across a triangle enclosure. Malleswaram et al. [5] analyzed numerically mixed convection with MHD in a lid-driven cavity with a corner heater. N. A. Bakar et al. [6] examined the dominance of a magnetic field on combined convective heat transmission in a square lid-driven enclosure. K. M. Gangawane et al. [7] simulated the impact of block position on mixed convection inside a lid-driven enclosure including a three-sided block with consistent heat flux. Saha et al. [8] executed a numerical analysis on the influence of inner heat production or absorption on MHD mixed convective flow inside a lid-driven compartment. Al-Amiri et al. [9] examined mixed convective flow within a lid-driven enclosure including a sinusoidal undulating lowest surface.

Asad et al. [10] published a numerical investigation of natural convective circulation within a hexagonal enclosure including a vertical fin. The mixed convection of MHD flow within a Nano fluid-filled, partly heated, undulating sidewall lid-driven enclosure was surveyed by Oztop et al. [11]. Asad et al. [12] published a discussion on the influence of waves on magneto-free convective heat transportation in an enclosure having vertically wavy sides. Aswatha et al. [13] have recorded the effect of different thermal boundary circumstances at the lower wall on free convection in enclosures. Natural convection within a porous compartment on both sidewalls, including sinusoidally heated, was concluded by S. Sivananand and M. Bhuvaneswari [14]. H.t. Cheong [15] noted the consequence of wall inclination on free convection inside a porous trapezoidal cavity. Das and Mahmud [16] numerically examined free convection within an enclosure involving two isothermal parallel undulating walls also two adiabatic vertical straight walls. Sarris et al. [17] have experimented with free convection within a 2D enclosure having sinusoidal upper wall temperature.

Based on the review of the above writing, mixed convection inside the enclosure was analyzed. A numerical investigation will be conducted at different Richardson numbers in presence of heat absorption.
and generation on a cavity having a wavy bottom wall.

2. MODELING

A square cavity having wavy bottommost walls is constructed in this study. The dimension and boundary constraints are presented in Fig. 1. The cavity perpendicular walls are kept cold, the lower undulating wall surface of a cavity is heated, and the top wall is maintained at an adiabatic temperature. There is a magnetic field having magnitude $B_0$ in the reverse way of the moveable lid.

![Figure 1: The model with boundary conditions is schematically shown.](image)

According to Saha et al. [8], the dimensionless governing equations are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (2.1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \nabla^2 U \quad (2.2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\partial V}{\partial X} + Ri\theta - \frac{Ha^3 V}{Re} \quad (2.3)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\nabla^2 \theta}{Pr Re} + \frac{\Delta}{Pr Re} \theta \quad (2.4)$$

Where,

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0},$$

$$P = \frac{p}{\rho U_0^2} \quad (2.5)$$

Prandtl number $Pr$, Reynolds number $Re$, Grashof number $Gr$, Richardson number $Ri$, and Hartmann number $Ha$ are the dimensionless parameters that occur in the equations. The following are the relative definitions of them:

$$Re = \frac{U_0 L}{\nu}, \quad Pr = \frac{v}{\nu}, \quad Ha^2 = \frac{\sigma B_0^2 L^2}{\mu},$$

$$Gr = \frac{g \beta \Delta T L^3}{\nu^2} \quad \text{and} \quad Ri = \frac{Gr}{Re^2} \quad (2.6)$$

where $\Delta = \frac{\rho_0 L^2}{\kappa \rho_0 C_p}$ is the heat absorption or production coefficient, $\nu$ is the kinematic viscosity, $\kappa$ denotes thermal conductivity and $C_p$ denotes specific heat.

Dimensionless boundary conditions:

On the vertical walls: $U = V = 0, \theta = 0$ \quad (2.7a)

On the upper horizontal wall:

$$U = 1, V = \frac{\partial \theta}{\partial Y} = 0 \quad (2.7b)$$

On the lower wavy surface:

$$U = V = 0, \theta = 1 \quad \text{for} \quad Y = A(1 - \cos 2\pi x), \quad 0 \leq X \leq 1. \quad (2.7c)$$

According to Saha et al. [8], On the basis of the non-dimensional variables, we can write down the moderate Nusselt number at the enclosure’s left vertical wall as

$$Nu_{av} = - \int_0^1 \frac{\partial \theta}{\partial n} \bigg|_{wall} \quad (2.8)$$

3. MATHEMATICAL TECHNIQUE

The dimensionless equations (2.1) to (2.4) towards the boundary constraints (2.7a) to (2.7c) are resolved by employing the Galerkin weighted residual approach of the finite element scheme. This numerical analysis employed six nodes with free triangular components. The confluence measure for the solution procedure has been selected as $\left(\|w^m - w^n\|^2\right)^{1/2} \leq 10^{-5}$ where $m$ is the amount of iterations with function of $U, V, \theta$. The formulation of this approach and computational methodology is well depicted by Asad et al. [19].

Validation:

The precision of the numerical outcomes obtained in this study is validated against the numerical investigations of Basak et al. [18]. Our current scheme is congruent with those of Basak et al. [18] in the nonexistence of $Ha = 0$ and heat production or absorption $(\Delta = 0)$ influences. The comparability for temperature contours is shown in Figure 2 for $Re = 1$ and $Gr = 10^4$ and $10^5$. It is obvious that a complete agreement exists between both the recent analytical simulation and those of Basak et al. [18].
Grid test:
To choose a suitable grid size for the current analysis at $Pr = 0.70, \lambda = 2, Ha = 10, Ri = 1$ and $\Delta = 5$. A grid refinement test was evaluated with five varieties of meshes, moderate Nusselt number on the cold left wall is found.

**Table-1:** Grid testing for $Nu_{\text{avg}}$ at $\lambda = 2, Ri = 1$ and $\Delta = 5$.

<table>
<thead>
<tr>
<th>Grid size</th>
<th>Nodes</th>
<th>Elements</th>
<th>$Nu_{\text{avg}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid-1</td>
<td>419</td>
<td>750</td>
<td>4.8982486</td>
</tr>
<tr>
<td>Grid-2</td>
<td>574</td>
<td>1047</td>
<td>4.8739823</td>
</tr>
<tr>
<td>Grid-3</td>
<td>1050</td>
<td>1966</td>
<td>5.0299615</td>
</tr>
<tr>
<td>Grid-4</td>
<td>3194</td>
<td>6166</td>
<td>5.0940569</td>
</tr>
<tr>
<td>Grid-5</td>
<td>12237</td>
<td>24040</td>
<td>5.0940570</td>
</tr>
</tbody>
</table>

The grid size of 3194 nodes and 6166 elements provided a suitable approach for the existent numeric analysis, as shown in the table.

4. RESULTS AND DISCUSSION
A numerical analysis using the finite element technique was carried out to assess the influences of inner heat generation or absorption on laminar combined convective heat transfer and flow pattern in a lid-driven enclosure through a wavy bottom wall. Outcomes are illustrated via isotherms and streamline accompanied by essential graphs at the two distinct internal temperatures $\Delta = -5$ and 5 along with undulation $\lambda = 2$, amplitude $A = 0.1$, Hartman number $Ha = 10$, Reynolds number $Re = 100$ and Prandtl number $Pr = 0.70$ for distinct values of Richardson number $Ri = 1$ to 10. Again, the numerical significance of rate of heat removal in phrases of $Nu_{\text{avg}}$ and the moderate temperature fluid $\theta_{\text{avg}}$ are also displayed in tabular form.

**Figure 3** exhibited the enactment of Richardson number in the existence of heat absorption and generation $\Delta = -5$ and 5 via streamline contours by considering $Re = 100$. In fact, the analysis is performed at Richardson number $Ri = 1, 5, 10$ and the corresponding value of Reynolds number $Re = 100$.

From **Figure 3**, we scrutinized that the velocity inside the enclosure advanced with advancing the value of a Richardson number. In the subject of heat absorption($\Delta = -5$) when $Ri = 1$ there are two vortices inside the top of the cavity. The clockwise-turning primary vortex generated by the moveable lid is prevalent and the minor vortex is at the tip of the left vertical edge of the enclosure. For $Ri = 5$, The buoyancy force within the enclosure is momentous, and three vortices look like inside the enclosure. At the same time, the strength of the crucial vortex as well as the secondary vortices is also enriched. The flow structure becomes more significant as the Richardson numbers rise to $Ri = 10$. On the other hand, for heat generation the streamlines pattern almost same as previous case but the cell distance and number of cells increased compare to the previous case. Which implies the fluid velocity increases in case of heat generation compare to absorption.
In figure 4 we represent the impact of the Richardson number (1 ≤ Ri ≤ 10) in the fact of heat generation and absorption (−5 ≤ Δ ≤ 5). Figure 4 characterizes that the isotherms within the enclosure are discovered to undergo the same transformations due to the impact in the case of inner heat absorption Δ = −6, where the heated area exchanges more immediately with the heated wall. A reduced amount of energy is detected to be brought away from the moving upper wall to the enclosure and, henceforth, As a result of the conveyance heat transfer governance being the heat line dominant of energy exchange within the enclosure, a powerful thermal boundary layer has formed towards the bottom. It may be commented that the isotherm contour is raised toward the top insulated wall and there is a temperature rise for Ri = 5. Which implies heat convection increased with increasing Ri. In the case Ri = 10, isotherm pattern almost same as Ri = 5 but number of extended isotherm contour and temperature gradient increased. There is no significant change in isotherm pattern due to heat generation except the increased in the number of distorted isotherm contours and temperature gradient.

![Figure 4: Isotherm variations for 1 ≤ Ri ≤ 10 at Δ = -5 and Δ = 5.](image)

### Heat transfer rate

Table 2 and Table 3 depict the numerical significance of the mean Nusselt number toward the hot wavy surface and mean fluid temperature inside cavity with advancing the value in Richardson number (1 ≤ Ri ≤ 10) for heat absorption and generation (−5 ≤ Δ ≤ 5) and Pr = 0.70 while the residual parameters are fixed.

Table 2 describes the fluctuation of mean Nusselt number Nu<sub>avg</sub> with the several Ri on the left vertical wall. It observed that Nu<sub>avg</sub> improves with the improve in Ri in both cases (heat absorption and heat generation). For a selected Ri, the moderate Nusselt number also improves with an improves of temperature.

**Table 2: Numerical significances of mean Nusselt number against on the bottom heated wall for specified value of Δ at Pr = 0.70, λ = 1, Ha = 20, Re = 100.**

<table>
<thead>
<tr>
<th>Ri</th>
<th>Mean Nusselt number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δ = -5</td>
</tr>
<tr>
<td>1</td>
<td>5.0702</td>
</tr>
<tr>
<td>5</td>
<td>5.3791</td>
</tr>
<tr>
<td>10</td>
<td>5.4674</td>
</tr>
</tbody>
</table>

Moreover, according to Table 2, the maximum Nu<sub>avg</sub> is 5.5605 on the left cold wall obtained for the heat generation Δ = 5 and Ri = 10.

Table 2 demonstrates the fluctuation of the moderate fluid temperature (θ<sub>avg</sub>) with the variation in Ri on the left vertical wall. It was marked that θ<sub>avg</sub> enlargements with the boost in Ri in both cases (heat absorption and heat generation). For a specified Ri, the moderate fluid temperature inside the cavity also enriches with in case of heat generation. From Table 3, we obtained the optimum fluid temperature 2.0386 at Δ = 5 and Ri = 10.

**Table 3: Numerical significances of moderate fluid temperature against on the bottom heated wall for specified value of Δ while Pr = 0.70, λ = 1, Ha = 20, Re = 100.**

<table>
<thead>
<tr>
<th>Ri</th>
<th>Moderate Fluid Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δ = -5</td>
</tr>
<tr>
<td>1</td>
<td>1.8480</td>
</tr>
<tr>
<td>5</td>
<td>1.9727</td>
</tr>
<tr>
<td>10</td>
<td>2.0371</td>
</tr>
</tbody>
</table>
5. CONCLUSION

The following are some of the most significant takeaways from this research:

- The Richardson number has a consequential impact on the flow and temperature field. The velocity of the fluid enhanced with raise of Richardson number.
- In both cases (heat absorption and generation), the moderate Nusselt number and moderate fluid temperature enriched with the boost in $Ri$. The optimum result is found at $\Delta = 5$ and $Ri = 10$.
- The heat transmission implementation and fluid temperature enhance with the improvement from absorption to generation.

REFERENCES