A STRUCTURAL ANALYSIS OF STRESS CONCENTRATION FACTOR OF RECTANGULAR OPENINGS IN SHIPS WITH VARIOUS CORNER RADII

Musih Mahfuza Mukta¹, Souptik Roy², Md. Shahidul Islam³

Department of Naval Architecture and Marine Engineering, BUET, Dhaka, Bangladesh
¹E-mail: Mukta12023@gmail.com
²E-mail: souptik_95@gmail.com
³E-mail: shahid7777@name.buet.ac.bd

ABSTRACT

Plate discontinuities are a typical occurrence in ships. Such common discontinuities include hatch openings on decks, doors, and windows on superstructures, sea inlets and bow thrusters on hull sides, lighting holes on floors, manholes, etc. Most of the time, these openings are rectangular, circular, and elliptical in shape. The stress is concentrated near the apertures because ships are always under stress during their operation. If the corners of the openings are sharp, the stress concentration increases dramatically, making it more likely for the concentrated energy to be released by crack propagation. A lot of research has been done on circular and elliptical openings. As a few studies on rectangular openings have been found, the stress concentration factor on rectangular openings for various corner radii has been chosen as the topic. Four parameters have been considered in this research: opening size, corner radius, opening orientation, and plate dimension. A lot of models, considering different corner radii for different opening size and orientation, have been assessed by renowned Finite Element software ABAQUS. The results have been shown by analyzing ABAQUS results and reports and preparing tables and graphs. The von-mises stress is analyzed mainly at nodal points for the most vulnerable regions of different openings. The maximum stress concentration for each model has been used for comparison. As an inclined opening could lead to a higher stress concentration factor, it is also investigated in this research. Finding the optimal shape of the opening for a particular geometric configuration is the target of this study which has been successfully achieved. The authors believe this study will be a guide for Naval Architects in designing ships ensuring safety more accurately.

Keywords: Rectangular opening, Von mises stress, Stress concentration factor.

1. INTRODUCTION

The innovation and creativity of a new product occurs mostly at the conceptual design stage and the analysis and synthesis at the detailed design stage is most likely the routine design activities. However, such an analysis is very time-consuming and not trivial due to customized geometries, features, and dimensions of objects, as well as the complexity of loads. It is desirable to have some guides and methods that can be adopted to correlate external loads and geometric features of an object to the system behaviors directly. In this way, the effects of external loads on the geometric features of objects can be quantified and utilized to optimize products. In the theory of elasticity, the stress concentration factors (SCFs) method is widely adopted to analyze the stresses for the prescribed geometries under given loading conditions. Now a question is arising about the stress concentration factor. So, what is stress concentration factor and where does it exist? A body which is under loading, stress can be concentrated near the discontinuities of the body. In this thesis, we are going to discuss about stress concentration of a perforated deck plates in a ship with various corner radii. When an opening penetrates the strength deck of a ship, the flow of stress in the deck is altered such that concentrations of stress form near the opening. Stress concentrations are of concern during both the preliminary and detail design and need to be addressed from the perspectives of strength. Openings in the hull structure constitute a discontinuity that can result in fatigue cracking. Unfortunately, ships require openings in the structure to satisfy their function. Examples of discontinuities that need be considered are hatches in decks, piping openings in decks and bulkheads, ventilation openings in structure, openings for through hull fittings, etc. The cyclic loading incurred from the hogging and sagging of the ship as it encounters wave causes this fatigue cracking. So, it is important for optimal design to consider the stress concentration factor.

1.1 Literature Review

Stress concentration in a perforated plate is a very popular topic with researchers. They try to compensate or diminish stress concentration as much as possible. But unfortunately, it is not possible to eliminate the
whole stress concentrations as the plate is not totally intact. Only in intact plates, stress is distributed over the whole plate and stress can be used totally. A lot of studies and research has been carried out to study the stress distribution in plates containing holes or openings. Different methods such as numerical, experimental, and analytic methods were used to analyze the stress distribution on plates. Again, studies regarding stress concentration factor in a perforated plate are also found.

A study from Chauhan et al. [1], analyzes stresses in finite anisotropic plate with a rectangular hole. In this study, complex variable approach is used for analysis. The result of this study is useful to predict the effect of material properties, hole geometry, plate size and loading conditions on stress distribution around the rectangular cutouts. Moreover, this result of the study is capable to produce a satisfactory solution for infinite plate by considering the large plate size.

Pan et al. [6] showed a study on solving the stress distribution problem of an infinite plate with a rectangular hole under uniaxial tension loading. They had done the analysis using a common analysis tool ANSYS. A good agreement is found in this study.

The effects of hole sizes, hole orientations and plates' aspect ratio on the stress distribution and stress concentration factor in a finite plate with a rectangular hole are considered. The results show that the amplitude values of stresses around the hole increases as the hole size increases. Moreover, the SCF firstly increases as hole orientation increases and then decreases as hole orientation increases. Again, the SCF decreases as the plate’s aspect ratio increases from 1 to 3 and increases as plate’s aspect ratio decreases from 1 to 0.25.

Sharma [7] presented the stress field around polygonal shaped cutouts in infinite isotropic plates. Here, effect of corner radius of polygonal cutouts on SCF is found. By controlling corner radius of the polygonal holes, the SCF can be kept within some permissible limit. As the corner radii approaches zero, SCF approaches infinity.

A general solution for stress concentration of an infinite plate with an elliptical hole under biaxial loading is presented by Xin-Lin Gao [8]. With two adjustable parameters – the biaxial loading factor and orientation angle of the hole – were considered for the study. This solution has become the most general solution for elliptical holes.

A study done by Yang et al. [9], shows an analytical solution for stress concentration problem of an infinite plate with a rectangular hole under biaxial tension. For this study a finite element governing equation has been established to analyze. In this study, the parameters of rectangular cutouts have been changed and different types of results are found.

From all these reviews, analysis on stress distribution in a plate with or without openings are available. For cutouts like circular, elliptical, polygonal (hexagonal and square) openings are considered. Analysis on rectangular openings is found rarely. Moreover, in those studies the corner radii of the rectangular openings are not considered as a parameter of the study. Another study found about corner radii of polygonal holes, the SCF can be kept within some permissible limit. As the corner radii approaches zero, SCF approaches infinity.

Ozkan et al. [5] presented a study of stress concentration factor in a plate with a circular hole with five different model solutions. These models are empirical model, REGA, ANN, analytical model, and FEA. In this study, some deviations are found between empirical solution and data obtained by using other four model solutions. The analytical and the FEA models have shown some deviations when d/w (d = diameter of hole, w = width of the plate) ratio is larger than 0.042. They also applied regression model analysis to be sure if the results are accurate or not.

Ogonowski [4] had a study of stress concentration in finite geometry plates. A lot of stress distribution predictions in finite plates are found in this study. In this study, an analytical technique is applied to predict stresses in finite geometry. Doubly connected anisotropic plates with stress concentration has been presented and verified with known solutions.

Ogandair et al. [3] shows the effect of material property variation on the SCF due to a circular hole in functionally graded panels. This study presented that a desired reduction in SCF is observed when the material property (like Young’s modulus) progressively increased away from the center of the hole.

Jafari et al. [2], showing a study of an analytical solution, employed a method to analyze stress distribution in plates with different shapes of regular holes. The results presented that due to uniaxial loading, the stress concentration factor can be significantly varied by changing the hole shape bluntness and rotation angle of hole. In finite plate containing square and hexagonal cutouts for a wide range of bluntness, the desirable SCF are less than the desirable SCF of similar plates with a circular hole.

Infinite plates with an elliptical hole under biaxial loading, the stress concentration problem of an infinite plate with a rectangular hole under biaxial tension. For this study a finite element governing equation has been established to analyze. In this study, the parameters of rectangular cutouts have been changed and different types of results are found.

2. OBJECTIVES

- Calculating the Stress Concentration Factor (SCF) of rectangular openings for different corner radii and different width ratio in a finite plate under uniaxial loading.
- Developing stress concentration factor vs r/b ratio curve (where r= corner radius b = opening width)
- Determining the best opening geometry.
• Investigation of the behavior of the plate if the opening is inclined.

3. THEORETICAL APPROACH

For pages other than the first page, start at the top of the page, and continue in double-column format. The two columns on the last page should be as close to equal length as possible. A fundamental case of stress concentration is the stress distribution around a circular hole in an infinite thin element (panel) subjected to uniaxial in-plane tension. In polar coordinates, Timoshenko and Goodier treated it as a plane stress problem, with the applied stress $\sigma$ in the theory of elasticity

$$
\sigma_r = \frac{1}{2} \sigma \left( 1 - \frac{a^2}{r^2} \right) + \frac{1}{2} \sigma \left( 1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4} \right) \cos 2\theta
$$

$$
\sigma_\theta = \frac{1}{2} \sigma \left( 1 + \frac{a^2}{r^2} \right) - \frac{1}{2} \sigma \left( 1 + \frac{3a^4}{r^4} \right) \cos 2\theta
$$

$$
\tau_{r\theta} = -\frac{1}{2} \sigma \left( 1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4} \right) \sin 2\theta
$$

where $a$ is the radius of the hole, $r$ and $\theta$ are the polar coordinates of a point in the element as shown in Fig. 1. At the edge of the hole with $r = a$,

$$
\sigma_r = 0
$$

$$
\sigma_\theta = \sigma \left( 1 - 2 \cos 2\theta \right)
$$

$$
\tau_{r\theta} = 0
$$

![Figure 3.1: Infinite thin element with hole under tensile load.](image)

At point, $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ then

$$
\sigma_{B\theta} = 3\sigma
$$

This is the maximum stress around the circle, so the SCF for this case is 3. The hole in a panel is such a commonly referenced case that often other SCFs are compared to the ‘standard’ of 3.

The distribution of $\sigma_\theta$ at the edge of the hole is shown in Fig. 2. At point $B$, with $\theta = 0$, we get,

$$
\sigma_{B\theta} = -\sigma
$$

At point, $\theta = \pm \frac{\pi}{6}$ or $\pm \frac{5\pi}{3}$ then

$$
\sigma_\theta = 0
$$

![Figure 3.2: Circumferential stress distribution on the edge of a circular hole.](image)

Considering section I-I, which passes through the center of the hole and point $A$, as shown in Fig. 1. For the points on section I-I, $\theta = \frac{\pi}{2}$ (or $3\pi/2$) and Eq. (1) becomes

$$
\sigma_r = \frac{3}{2} \sigma \left( 1 - \frac{a^2}{r^2} \right)
$$

$$
\sigma_\theta = \frac{1}{2} \sigma \left( 1 + \frac{3a^4}{r^4} \right)
$$

$$
\tau_{r\theta} = 0
$$

From Eq. (3), it can be observed that on cross section I-I, when $r = a$, $\sigma_\theta = 3\sigma$, and as $r$ increases, $\sigma_\theta$ decreases. Eventually, when $r$ is large enough, $\sigma_\theta = \sigma$, and the stress distribution recovers to a uniform state. Also, it follows from Eq. (3) that the stress concentration caused by a single hole is localized. When, for example, $r = 5.0a$, $\sigma_\theta$ decreases to 1.02$\sigma$. Thus, after $5a$ distance from the center, the stress is very close to a uniform distribution.

The stress distribution over cross section II-II of Fig. 1 can be obtained using similar reasoning. Thus, from Eq. (1) with $\theta = 0$ (or $\theta = \pi$),

$$
\sigma_r = \frac{1}{2} \sigma \left( 2 - \frac{5a^2}{r^2} + \frac{3a^4}{r^4} \right)
$$

$$
\sigma_\theta = \frac{1}{2} \sigma \left( 1 + \frac{3a^4}{r^4} \right)
$$

$$
\tau_{r\theta} = 0
$$

4. MODEL ANALYSIS & RESULT

4.1 Finite Element Analysis for Different Models

Abaqus CAE version 6.13 has been used for this analysis. In this section, different analysis for two-dimensional plate model has been done. Different openings in those plates have been shown.

The result has been showed by taking image of Abaqus and making table and graphs. The von-mises
stress is analyzed mainly at nodal points for the most vulnerable regions of different openings.

Basically, the maximum stress concentration for each model has been used for comparison. The main study is to figure out what could be the greatest opening geometry. Several models have been created and analyzed for this objective. Changes in plate dimensions, opening size, and opening orientation have been used to create those models. Several tables and graphs have been created based on those models to compare the stress concentration factor. An optimal opening shape has been determined by determining the lowest feasible stress concentration factor.

4.2 Analysis

For the analysis, the model is divided into 4 quadrilateral elements.

The useful quantities used in those models are:

- Applied Uniform pressure has been used for all the model is \( P = 1000 \) N. Assuming thickness of the plate is \( t = 10 \) mm, Young’s Modulus, \( Y = 210 \) GPa, Poisson’s ratio, \( \nu = 0.33 \), Plate length, \( L = 6 \) m, Plate width, \( B = 2 \) m.

The boundary conditions and loadings are applied using the ABAQUS software as shown in the figure given below (4.1). In this figure. The nodes of the bottom edge of model have been considered as X axis. Those nodes can move along the x-axis and are constrained in the y direction (\( U_y = 0 \)). The vertical nodes considering as Y axis and are constrained along the x direction (\( U_x = 0 \)).

![Figure 4.1: Rectangular plate with boundary conditions](image)

Here, also the Grid independence test is done. The 1st attempt is making comparison between the Rounded, Rectangular, and Sharp edge notch.

<table>
<thead>
<tr>
<th>Opening type</th>
<th>Maximum Local Stress (Pa)</th>
<th>Stress concentration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounded</td>
<td>3209</td>
<td>3.209</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Rectangular</th>
<th>Sharp edge with vertical notch</th>
<th>Sharp edge with horizontal notch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress concentration</td>
<td>6805</td>
<td>16890</td>
<td>10660</td>
</tr>
<tr>
<td>factor</td>
<td>6.805</td>
<td>16.89</td>
<td>10.66</td>
</tr>
</tbody>
</table>

This can be seen that generating any kind of sharp notch is the worst case. The most favorable condition can be achieved by designing rounded openings which stress concentration factor is minimum among those opening type.

Thus, the further study of this study is to make detail table of stress concentration factor for various type of rounded opening.

An example of the analysis is given below (Figure 4.3). The plate dimension of this model is based on when opening length (\( a \)) is equal to twice of opening width (\( b \)).

![Figure 4.2: Result in terms of von mises stress.](image)

The generated graph with various r/b ratio for this model is given below. (Figure 4.3)

![Figure 4.3: Comparison graph for the various r/b ratio.](image)

Another example is given below of a model where the opening length (\( a \)) is equal to twice of opening width (\( b \)), but load is working diagonally with the opening.

![Figure 4.4: Results in terms of von mises stress of this model.](image)
The comparison graph for stress concentration factor to r/b ratio is given below when pressure is applied diagonally and opening length is equal to twice of opening width (a=2b).

![Comparison graph for stress concentration factor to r/b ratio](image)

Figure 4.5: Comparison graph for the various r/b ratio

Similarly, lots of model changing the plate dimension and orientation have been done.

### Table-4.2: Comparison of different opening geometry of rounded opening

<table>
<thead>
<tr>
<th>a:b ratio</th>
<th>Maximum concentrated stress (Pa)</th>
<th>Stress concentration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>4110</td>
<td>4.11</td>
</tr>
<tr>
<td>2:1</td>
<td>3185</td>
<td>3.185</td>
</tr>
<tr>
<td>3:1</td>
<td>3364</td>
<td>3.364</td>
</tr>
<tr>
<td>1:3</td>
<td>8262</td>
<td>8.262</td>
</tr>
<tr>
<td>1:2</td>
<td>5238</td>
<td>5.238</td>
</tr>
<tr>
<td>1:1 (inclined), 45°</td>
<td>4458</td>
<td>4.458</td>
</tr>
<tr>
<td>2:1 (inclined), 45°</td>
<td>5423</td>
<td>5.423</td>
</tr>
</tbody>
</table>

Again, with fixed corner radius, another comparison table has been made which is given below.

### Table-4.3: Comparison of different opening geometry of rounded opening considering constant corner radius.

<table>
<thead>
<tr>
<th>a:b ratio</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>3.324</td>
</tr>
<tr>
<td>2:1</td>
<td>3.158</td>
</tr>
<tr>
<td>3:1</td>
<td>3.040</td>
</tr>
<tr>
<td>1:3</td>
<td>8.150</td>
</tr>
<tr>
<td>1:2</td>
<td>4.596</td>
</tr>
<tr>
<td>1:1 (inclined), 45°</td>
<td>3.300</td>
</tr>
<tr>
<td>2:1 (inclined), 45°</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Here, there is also made some models with changing plate dimension. For all those models taking fixed corner radius, r= 0.25 m.

### Table-4.4: Comparison of constant opening geometry for different plate size

<table>
<thead>
<tr>
<th>Plate length × Width (m²)</th>
<th>Plate length to width ratio</th>
<th>Max stress concentration (Pa)</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x1</td>
<td>5:1</td>
<td>3295</td>
<td>3.29 5</td>
</tr>
<tr>
<td>4x1</td>
<td>4:1</td>
<td>3303</td>
<td>3.30 3</td>
</tr>
<tr>
<td>6x2</td>
<td>3:1</td>
<td>3040</td>
<td>3.04 0</td>
</tr>
<tr>
<td>4x2</td>
<td>2:1</td>
<td>2525</td>
<td>2.52 5</td>
</tr>
</tbody>
</table>

5. CONCLUSION

5.1 The following is a summary of the study's findings:

A ship must be subjected to cyclic loads such as hogging and sagging. As a result, the stress will be concentrated. It is important to reduce the stress.

A decision can be made from data table-4.2 that the concentrated stress is minimized when the opening is rectangular (a=2b). This concentrated stress cannot be eliminated in any way because there's no way to build an opening that doesn't generate stress. Moreover, if there is an opening with a length equal to three times the breadth, the concentrated stress can be extremely high.

Thus, for a newly designed ship, the opening height must be reduced in proportion to the opening length.

Among all generated models, it is found that the most ideal situation is with an opening length of 3 times of the opening width and a corner radius to opening width ratio of 0.25. For this model, the stress concentration factor is 3.040. (From table-4.3)

It is observed that when the opening length increases in relation to the opening width, the local stress as well as the stress concentration factor decreases.

An inclined opening could lead to a higher stress concentration factor. From the comparison table 4.2, it is found that the lowest stress concentration factor for inclined opening is 4.458 where the a:b is 1:1. From table 4.3, the lowest stress concentration factor is found 3.300 where the a:b is 1:1 and inclined. These two values are comparatively higher than the value found for the regular rectangular opening geometry which has no inclination. As a result, the
optimal design solution should have horizontal rectangular opening geometry.

A conclusion can be deducted from the comparison table of changing plate dimensions (table 4.4) that when the plate length to breadth ratio falls, the stress concentration factor lowers as well. Different plate dimensions (5m×1m, 4m×1m, 6m×2m, 4m×2m) are shown with a fixed opening dimension of 1.8m×0.6m. Among all these models, the least stress concentration factor is found 3.040 for the plate dimension of 6m×2m.

Mainly, for a ship’s opening, the opening length must be greater than the opening width. Also, the opening length should be parallel to the plate length.

6. REFERENCES


